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RHEOLOGICAL EFFECTS IN ELASTIC
EXPULSION FROM LONG TUBES

BY



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A THESIS

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "Rheological Effects in Elastic Expulsion From Long Tubes" submitted by Lee F. Siebert in partial fulfilment of the requirements for the degree of Master of Science.

ABSTRACT

This thesis investigates the influence of rheology on elastic expulsion from a long pressurized tube following instantaneous severance. A differential equation is constructed for the inner radius of the tube as a function of axial position and time. The equation incorporates power law fluids and tubes for which the pressure-radius relationship takes either of two particular forms. Similarity solutions are developed for a wide range of conditions.

Physical experiments using a Newtonian fluid and latex tubes are discussed in detail and results presented.

Also included is a description of in vivo experiments on the canine venous system.

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NOMENCLATURE

r	radial distance from tube axis
X	axial distance from point of severance
L	tube length
R	inner radius of tube
S	area
V	volume
t	time
$\vec{v}$	velocity vector
$\vec{\sigma}$	stress vector
$\vec{G}$	body force per unit mass
P	static pressure
σ_{xr}	axial shear stress on the r surface
U	axial velocity
ϕ	normalized dilatation
A, B, C	coefficients in dilatation equation
f, g, F, I	functions defined in text
M, K	rheological parameters of tube wall
N	measure of non-Newtonian behavior
ρ	fluid density
μ	fluid viscosity
λ	nominal viscosity
ν	kinematic viscosity (μ/ρ)
α, β	rheological parameters

NOMENCLATURE (continued)

Subscripts

P	pressure limited
R	radius limited
0	relaxed
1	fully extended

Superscripts

'	denotes differentiation with respect to η unless designated otherwise
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CHAPTER I

INTRODUCTION

1.1 Theory

The presence and role of tubes in length much greater than their diameter in animals and plants has been noted by such investigators as Frey-Wyssling [18][#], and Johnson, Smith and Lock [17]. Examples include lacticifiers of trees, nerve fibers and blood vessels. Recently, Lock [1] carried out a comprehensive mathematical analysis of the elastic expulsion of a Newtonian fluid from a long tube with a wall pressure-radius relationship characterized by a linear function. This thesis is an extension of that work and develops a mathematical model applicable to power law fluids and wall pressure-radius relationships represented by an exponential function; the Newtonian fluid and linear wall constitute special cases. The exponential function used takes on two forms, one for a yielding material, the other for a locking material. The resulting equations for both cases are discussed with respect to their rheological parameters.

This study considers the flow history from a point in time when the intact vessel is suddenly severed. Under these conditions, fluid flows from the tube under the influence of a pressure gradient caused partly by the maintenance of a sufficient upstream pressure and partly by contraction of the initially distended wall. If the fluid is initially at

[#]Numbers in square brackets refer to references listed on page 87.

rest, or flowing very slowly, the latter effect dominates, and it is this situation which will be considered here.

The problem is posed as the axi-symmetric flow of an incompressible fluid from a long, uniform, axially-unrestrained tube in which the pressure is initially uniform at such a value that the vessel walls are distended. After complete and instantaneous severance of the vessel, a flow system is set up in which the tension in the vessel wall maintains a driving pressure field. The effect of this field is countered predominantly by frictional effects within the fluid as generated by adherence of the fluid to the vessel wall.

1.2 Physical Experiments

Experimental studies of the elastic expulsion of a Newtonian fluid from a long tube of yielding material are described in detail. These physical experiments used thin-walled latex rubber tubes and glycerine-water solutions. Results compared favorably with those predicted by the theory.

1.3 Physiological Experiments

In recent years both engineers and medical doctors have shown a greater interest in the physical characteristics of the cardiovascular system, [8,9,10]. The development of mathematical and physical models provides another avenue for better understanding the behavior of the cardiovascular system. However, the majority of researchers have concentrated their efforts on the arterial system and the microvasculature; relatively little work has been reported dealing with the venous side

of the vascular tree. To date there have been no reports of in vivo studies of the elastic expulsion phenomenon in veins. Thus, from the experiments discussed herein it was hoped to make a start in this direction. The objectives were to conduct a pilot study in the hopes of obtaining qualitative, if not quantitative, information about elastic expulsion in veins, and the problems encountered in performing such experiments. These objectives were realized.

CHAPTER II

THEORY

2.1 Governing Equations

The development of an equation to represent elastic expulsion from a long tube [1] begins with the equations of continuity and momentum. The momentum equation for an incompressible continuum of volume $V(t)$ bounded by a simple surface $S(t)$ may be written

$$\frac{d}{dt} \iiint_{V(t)} \tilde{v} \, dV = \frac{1}{\rho} \iint_{S(t)} \tilde{\sigma} dS + \iiint_{V(t)} \tilde{G} \, dV \quad (1)$$

A dimensional analysis of the Navier-Stokes equation reveals that for the dimensional group $\nu t/R_1^2 \gg 1$, the inertia terms are negligible and the flow is dominated by normal and shear forces. Under such conditions and in the absence of body forces, the first and last terms of equation (1) become negligible and the problem is quasistatic in form. Using the co-ordinate system shown in Figure 2.1 and considering the flow in tubes of length much greater than diameter, that is capillaries, equation (1) reduces to the "Poiseuillian" form

$$0 = -\frac{1}{\rho} \frac{\partial P}{\partial X} + \frac{1}{r} \frac{\partial}{\partial r} (r \sigma_{xr}) \quad (2)$$

$$0 = -\frac{\partial P}{\partial r}$$

The solution of the equations (2) for the velocity field may be expressed as

$$U(r,X,t) = - f\left(\frac{\partial P}{\partial X}\right) g\left(R, \frac{r}{R}, \alpha_1, \dots, \alpha_m\right) \quad (3)$$

providing that the shear stress is a homogeneous function of the velocity gradient. The rheology of the fluid, upon which the solution depends, is expressed by the rheological parameters $\alpha_1, \dots, \alpha_m$.

The continuity equation for an incompressible medium reduces to

$$\iiint_{S(t)} \underline{v} \cdot d\mathbf{S} = 0 \quad (4)$$

which, when applied to axi-symmetric flow in a capillary, becomes

$$\frac{\partial}{\partial X} \int_0^R U r dr + R \frac{\partial R}{\partial t} = 0 . \quad (5)$$

Substituting for U from (3) yields

$$\frac{\partial}{\partial X} (fR^2I) - R \frac{\partial R}{\partial t} = 0 \quad (6)$$

where

$$I(R, \alpha_1, \dots, \alpha_m) = \int_0^1 \frac{r}{R} g\left(R, \frac{r}{R}, \alpha_1, \dots, \alpha_m\right) d\left(\frac{r}{R}\right)$$

and thus reflects the dependence of the volumetric flow rate upon the rheological behavior of the fluid.

Quasi-static radial equilibrium demands a balance between the

normal stresses on the inside of the tube wall and the membrane stresses within the wall. In turn, these tangential stresses are related to the tangential strain, or dilatation, through a constitutive relation and consistent with the requirements that $\nu t/R_1^2 \gg 1$, $L/R_1 \gg 1$ and the tube is untethered. Therefore it is possible to write

$$P - P_0 = F[(R-R_0), R_0, \beta_1, \dots, \beta_m] , \quad (7)$$

as a fairly general relation representing radial equilibrium, where $\beta_1, \dots, \beta_m$ are rheological parameters of the tube wall. It is reasonable to expect that F will be a monotonically increasing function if the constitutive relation is independent of time.

2.2 The dilatation Equation

Equations (6) and (7) may now be used to construct a differential equation for the dilatation (in this case a contraction) of the capillary wall. Equation (6) gives

$$\frac{\partial}{\partial X} f\left(\frac{\partial P}{\partial X}\right) R^2 I + f\left(\frac{\partial P}{\partial X}\right) I \frac{\partial R^2}{\partial X} + R^2 f\left(\frac{\partial P}{\partial X}\right) \frac{\partial I}{\partial X} - R \frac{\partial R}{\partial t} = 0$$

from which

$$\frac{\partial^2 P}{\partial X^2} = \frac{1}{IRf'} \frac{\partial R}{\partial t} - \left(\frac{2f}{Rf'} + \frac{fI'}{f'I} \right) \frac{\partial R}{\partial X}$$

where the prime denotes differentiation with respect to R for I and $\partial P/\partial X$ for f .

Putting
$$f\left(\frac{\partial P}{\partial X}\right) = f_1\left(R, \frac{\partial R}{\partial X}\right)$$

and
$$f'\left(\frac{\partial P}{\partial X}\right) = f_2\left(R, \frac{\partial R}{\partial X}\right)$$

results in

$$\frac{\partial^2 P}{\partial X^2} = \frac{1}{I R f_2} \frac{\partial R}{\partial t} - \left(\frac{2 f_1}{R f_2} + \frac{f_1 I'}{I f_2} \right) \frac{\partial R}{\partial X} \quad (8)$$

$\partial^2 P/\partial X^2$ is evaluated by differentiating equation (7) twice. Thus we get

$$\frac{\partial P}{\partial X} = F' \frac{\partial R}{\partial X}$$

and
$$\frac{\partial^2 P}{\partial X^2} = F'' \left(\frac{\partial R}{\partial X} \right)^2 + F' \left(\frac{\partial^2 R}{\partial X^2} \right)$$

which, when substituted into equation (8), yields the "general" dilatation equation

$$\frac{\partial R}{\partial t} = A\left(R, \frac{\partial R}{\partial X}\right) \frac{\partial^2 R}{\partial X^2} + B\left(R, \frac{\partial R}{\partial X}\right) \left(\frac{\partial R}{\partial X} \right)^2 + C\left(R, \frac{\partial R}{\partial X}\right) \frac{\partial R}{\partial X} \quad (9)$$

where

$$A(R, \frac{\partial R}{\partial X}) = IRf_2F'$$

$$B(R, \frac{\partial R}{\partial X}) = IRf_2F''$$

$$C(R, \frac{\partial R}{\partial X}) = f_1(2I+RI')$$

This equation for tube radius is seen to be a second order equation in two independent variables. Its solution ensures the complete solution of the elastic expulsion problem posed. The pressure distribution follows immediately from equation (7) once $R(X,t)$ is known. The velocity field $U(r,X,t)$ and the volumetric flow rate $\dot{Q}$ may then be determined from equation (3).

The initial and boundary conditions which the solution must satisfy depend upon the precise shape of the tube and the manner in which the expulsion is allowed to take place. For the instantaneous severance of a long uniform tube they may be written as:

$$R(X,0) = R_1$$

$$R(0,t) = R_0 \tag{10}$$

$$R(\infty,t) = R_1$$

2.3 Power Law Fluids

The relationship between shear stress σ_{xr} and rate of shear strain for power law fluids may be expressed as

$$\sigma_{xr} = - \lambda \left(\frac{\partial U}{\partial r} \right)^N \quad (11)$$

where λ and N are constants for the particular fluid: λ is a measure of the nominal "viscosity" of the fluid; N is a measure of the degree of non-Newtonian behavior. $N < 1$ applies to pseudoplastic fluids and $N > 1$ to dilatant fluids. It should be noted that the dimensions of λ depends on the index N . Thus the ratio of shear stress to rate of shear strain, or apparent viscosity, μ_0 , is expressed by the relation

$$\mu_0 = \lambda \left| \frac{\partial U}{\partial r} \right|^{N-1} . \quad (12)$$

Considering the pressure acting on fluid in the tube

$$\frac{\partial P}{\partial X} \pi r^2 = 2\pi r \cdot \sigma_{xr}$$

from which

$$\sigma_{xr} = \frac{1}{2} \frac{\partial P}{\partial X} r .$$

Using this expression for σ_{xr} in equation (2) and integrating to obtain

an expression for the velocity field yields

$$U(r,X,t) = \frac{N}{N+1} \left(\frac{1}{2\lambda}\right)^{\frac{1}{N}} \left(\frac{\partial P}{\partial X}\right)^{\frac{1}{N}} R^{\frac{N+1}{N}} \left[1 - \frac{r^{\frac{N+1}{N}}}{R^{\frac{N+1}{N}}}\right] \quad (13)$$

which may be used to obtain an expression for I ; whence

$$I = \int_0^1 \frac{N}{N+1} \left(\frac{1}{2\lambda}\right)^{\frac{1}{N}} \left(\frac{r}{R}\right)^{\frac{N+1}{N}} R^{\frac{N+1}{N}} \left[1 - \frac{r^{\frac{N+1}{N}}}{R^{\frac{N+1}{N}}}\right] d\left(\frac{r}{R}\right) .$$

Integrating,

$$I = \left(\frac{1}{2\lambda}\right)^{\frac{1}{N}} \frac{N}{2(3N+1)} R^{\frac{N+1}{N}} \quad (14)$$

and

$$I' = \left(\frac{1}{2\lambda}\right)^{\frac{1}{N}} \frac{N+1}{2(3N+1)} R^{\frac{1}{N}} . \quad (15)$$

In addition

$$f_1 = f\left(\frac{\partial P}{\partial X}\right) = \left(\frac{\partial P}{\partial X}\right)^{\frac{1}{N}}$$

and

$$f_2 = f'\left(\frac{\partial P}{\partial X}\right) = \frac{1}{N} \left(\frac{\partial P}{\partial X}\right)^{\frac{1-N}{N}} \quad (16)$$

2.4 Wall Rheology

The materials considered in this thesis fall into two broad categories. The first can best be described as a yielding material which results in a pressure-limited case, Figure 2.2. The second category has properties which produce a radius limited case, and is best referred to as a locking material, Figure 2.3.

Together, these two cases cover a wide range of known materials, both physical and physiological, and their behavior can be represented approximately by an appropriate exponential function.

2.4.a Pressure Limited Case

For a yielding tube, equation (7) will be taken in the form

$$P - P_0 = K_P \left[1 - e^{-\frac{M_P}{R_0} (R - R_0)} \right] \quad (17)$$

where K_P is a constant with dimensions of pressure and M_P a dimensionless rheological parameter of the tube wall. Differentiating with respect to r ,

$$F' = \frac{\partial P}{\partial R} = \frac{K_P M_P}{R_0} e^{-\frac{M_P}{R_0} (R - R_0)} \quad (18)$$

and

$$F'' = \frac{\partial^2 P}{\partial R^2} = K_P \left(\frac{M_P}{R_0} \right)^2 e^{-\frac{M_P}{R_0} (R - R_0)} .$$

Equation (17) can also be used to evaluate the functions (16) as follows

$$f_1 = \left(\frac{\partial P}{\partial X}\right)^{\frac{1}{N}} = \left(\frac{\partial P}{\partial R} \frac{\partial R}{\partial X}\right)^{\frac{1}{N}} = \left[\frac{K_P M_P}{R_0} e^{-\frac{M_P}{R_0}(R-R_0)} \frac{\partial R}{\partial X}\right]^{\frac{1}{N}} \quad (19)$$

$$f_2 = \frac{1}{N} \left(\frac{\partial P}{\partial X}\right)^{\frac{1-N}{N}} = \frac{1}{N} \left(\frac{\partial P}{\partial R} \frac{\partial R}{\partial X}\right)^{\frac{1-N}{N}} = \frac{1}{N} \left[\frac{K_P M_P}{R_0} e^{-\frac{M_P}{R_0}(R-R_0)} \frac{\partial R}{\partial X}\right]^{\frac{1-N}{N}}.$$

The coefficients of the dilatation equation can now be evaluated for this material using equations (14), (15), (18) and (19). Thus,

$$A_P(R, \frac{\partial R}{\partial X}) = 'IRf_2$$

$$= \left(\frac{K_P M_P}{2R_0 \lambda}\right)^{\frac{1}{N}} e^{-\frac{M_P}{NR_0}(R-R_0)} \frac{\frac{2N+1}{N}}{\frac{R}{2(3N+1)}} \left(\frac{\partial R}{\partial X}\right)^{\frac{1-N}{N}}$$

$$B_P(R, \frac{\partial R}{\partial X}) = F''IRf_2$$

$$= -\frac{M_P}{R_0} \left(\frac{K_P M_P}{2R_0 \lambda}\right)^{\frac{1}{N}} e^{-\frac{M_P}{NR_0}(R-R_0)} \frac{\frac{2N+1}{N}}{\frac{R}{2(3N+1)}} \left(\frac{\partial R}{\partial X}\right)^{\frac{1-N}{N}}$$

$$C_P(R, \frac{\partial R}{\partial X}) = f_1(2I + RI')$$

$$= \left(\frac{K_P M_P}{2R_0 \lambda}\right)^{\frac{1}{N}} e^{-\frac{M_P}{NR_0}(R-R_0)} \frac{\frac{N+1}{N}}{\frac{R}{2}} \left(\frac{\partial R}{\partial X}\right)^{\frac{1}{N}}$$

Substituting these expressions into equation (9), multiplying through by

$$2(3N+1), \text{ dividing by } \left(\frac{K_P M_P}{2R_0 \lambda}\right)^{\frac{1}{N}} e^{-\frac{M_P}{NR_0}(R-R_0)}, \text{ and rearranging produces}$$

$$\begin{aligned}
2(3N+1) \left(\frac{2R_0\lambda}{K_P M_P} \right)^{\frac{1}{N}} e^{\frac{M_P}{NR_0} (R-R_0)} \frac{\partial R}{\partial t} &= R^{\frac{2N+1}{N}} \left(\frac{\partial R}{\partial X} \right)^{\frac{1-N}{N}} \frac{\partial^2 R}{\partial X^2} \\
&+ (3N+1 - \frac{M_P}{R_0} R) R^{\frac{N+1}{N}} \left(\frac{\partial R}{\partial X} \right)^{\frac{N+1}{N}} .
\end{aligned} \quad (20)$$

A search for similarity solutions produced

$$R - R_0 = (R_1 - R_0) \phi(\eta) ,$$

where

$$\eta = \frac{(R_1 - R_0)^{\frac{N-1}{N+1}}}{R_0^{\frac{2N+1}{N+1}}} \left(\frac{X}{t^{\frac{N}{N+1}}} \right) \left(\frac{2R_0\lambda}{K_P M_P} \right)^{\frac{1}{N+1}} \left[\frac{N(3N+1)}{(N+1)} \right]^{\frac{N}{N+1}} , \quad (21)$$

transforms equation (20) into an ordinary differential equation. This form of the dilatation equation reveals the dependence of the expulsion process on the rheological parameters M_P and N , as well as the dilatation ratio R_0/R_1 .

Thus, for a yielding material, the dilatation equation becomes

$$\begin{aligned}
&\left[\frac{R_0}{R_1} + \left(1 - \frac{R_0}{R_1} \right) \phi \right] \left[\frac{2N+1}{N} (\phi')^{\frac{1-N}{N}} \phi'' + \right. \\
&\left. [3N + 1 - M_P (1 + (\frac{R_1}{R_0} - 1)\phi)] \left[\frac{R_0}{R_1} + \left(1 - \frac{R_0}{R_1} \right) \phi \right]^{\frac{N+1}{N}} \left(1 - \frac{R_0}{R_1} \right) (\phi')^{\frac{N+1}{N}} \right. \\
&\left. + 2 \left(\frac{R_0}{R_1} \right)^{\frac{2N+1}{N}} e^{\frac{M_P}{NR_0} (R_1 - R_0)} \eta \phi' \right] = 0
\end{aligned} \quad (22)$$

with the boundary conditions as outlined in (10) transforming to

$$\phi(0) = \phi(\infty) - 1 = 0 .$$

Despite the rather formidable appearance of equation (22) it can be solved using an approach described elsewhere [2]. First, the equation is written formally as

$$a_{p1} \frac{1}{\phi'} \phi'' + a_{p2} \phi' + a_{p3} \eta = 0 \quad (23)$$

where the functions a_{p1} , a_{p2} and a_{p3} are given by:

$$\begin{aligned} a_{p1} &= \left[\frac{R_0}{R_1} + \left(1 - \frac{R_0}{R_1} \right) \phi \right]^{\frac{2N+1}{N}} (\phi')^{\frac{1}{N}} \\ a_{p2} &= [3N+1-M_p(1 + (\frac{R_1}{R_0} - 1)\phi)] \left[\frac{R_0}{R_1} + \left(1 - \frac{R_0}{R_1} \right) \phi \right]^{\frac{N+1}{N}} \left(1 - \frac{R_0}{R_1} \right) (\phi')^{\frac{1}{N}} \\ a_{p3} &= 2 \left(\frac{R_0}{R_1} \right)^{\frac{2N+1}{N}} e^{\frac{M_p}{N}} \left(\frac{R_1}{R_0} - 1 \right) \phi \phi' . \end{aligned} \quad (24)$$

Rearranging equation (23) slightly yields

$$\frac{1}{\phi'} \frac{d\phi'}{d\eta} = - \frac{a_{p2}}{a_{p1}} \phi' - \frac{a_{p3}}{a_{p1}} \eta . \quad (25)$$

Thus

$$\frac{1}{\phi'} \frac{d\phi'}{d\eta} = \frac{-\frac{R_0}{R_1} \phi'}{\left[\frac{R_0}{1-R_0/R_1} + \phi\right]} + \frac{M_P [1 + (R_1/R_0 - 1)\phi] \phi'}{\left[\frac{R_0}{1-R_0/R_1} + \phi\right]}$$

$$- \frac{2e^{\frac{M_P}{N} (R_1/R_0 - 1)\phi} \left(\frac{R_0}{R_1}\right)^{\frac{2N+1}{N}} \eta \cdot (\phi')^{\frac{N-1}{N}}}{\left[R_0/R_1 + (1-R_0/R_1)\phi\right]^{\frac{2N+1}{N}}}$$

which integrates to give

$$\phi'(\eta) = \frac{G_P e^{-F_P(\eta)}}{\left[\frac{R_0}{R_1} \left(\frac{1-R_0/R_1}{1-R_0/R_1} + \phi\right)^{3N+1}\right]} \quad (26)$$

where

$$F_P(\eta) = \int \left(\frac{2e^{\frac{M_P}{N} (R_1/R_0 - 1)\phi} \left(\frac{R_0}{R_1}\right)^{\frac{2N+1}{N}} (\phi')^{\frac{N-1}{N}} \eta}{\left[R_0/R_1 + (1-R_0/R_1)\phi\right]^{\frac{2N+1}{N}}} - \frac{M_P [1 + (R_1/R_0 - 1)\phi] \phi'}{\left[\frac{R_0}{1-R_0/R_1} + \phi\right]} \right) d\eta \quad (27)$$

and G is a constant of integration. Integrating once again:

$$\frac{1}{3N+2} \left[\frac{R_0}{1-R_0/R_1} + \phi\right]^{3N+2} = G_P \int e^{-F_P(\eta)} d\eta + H_P \quad (28)$$

where H_P is a second constant of integration. Now applying the boundary conditions:

(a) $\phi(0) = 0$

therefore

$$\frac{1}{3N+2} \left[\frac{R_0/R_1}{1-R_0/R_1} \right]^{3N+2} = H_p$$

(b) $\phi(\infty) = 1$

therefore

$$\frac{1}{3N+2} \left[\frac{R_0/R_1}{1-R_0/R_1} + 1 \right]^{3N+2} = G_p \int_0^\infty e^{-F(\eta)} d\eta + H_p$$

or

$$G_p = \frac{1}{3N+2} \left[\left(\frac{1}{1-R_0/R_1} \right)^{3N+2} - \left(\frac{R_0/R_1}{1-R_0/R_1} \right)^{3N+2} \right] \times \frac{1}{\int_0^\infty e^{-F_p(\eta)} d\eta} .$$

Equation (28) may now be written as

$$\phi(\eta) = [E_p(\eta) + \left(\frac{R_0/R_1}{1-R_0/R_1} \right)^{3N+2}]^{\frac{1}{3N+2}} - \frac{R_0/R_1}{1-R_0/R_1} \quad (29)$$

where

$$E_p(\eta) = \left[\frac{1 - (R_0/R_1)^{3N+2}}{(1-R_0/R_1)^{3N+2}} \right] \frac{\int_0^\eta \exp[-F_p(\eta)] d\xi}{\int_0^\infty \exp[-F_p(\eta)] d\xi}$$

and can be solved for $\phi(\eta)$ through the iterative process of successive substitutions.

The Fortran source program used to solve this equation numerically is outlined in Appendix E. Integration used a 3-point Simpson's Rule. Accuracy in $\phi(\eta)$ was controlled to obtain agreement of better than 10^{-5} between successive iterations. The program, like the equation, is constructed in a general form and is capable of determining values of $\phi(\eta)$ for any desirable combination of the parameters R_0/R_1 , N and M_p , which are represented in the program by DR, FR, and WR respectively.

2.4.b Radius Limited Case

A "locking" material may be represented

$$R - R_0 = K_R \left[1 - e^{-\frac{M_R}{P_0} (P - P_0)} \right] \quad (30)$$

where K_R is a constant with the dimensions of radius and M_R is a dimensionless rheological parameter of the tube wall. This equation may be used as in the previous section to show that

$$\begin{aligned} A_R(R, \frac{\partial R}{\partial X}) &= \left(\frac{P_0}{2M_R \lambda} \right)^{\frac{1}{N}} \left(\frac{1}{K_R - R + R_0} \right)^{\frac{1}{N}} \frac{R^{\frac{2N+1}{N}}}{2(3N+1)} \left(\frac{\partial R}{\partial X} \right)^{\frac{1-N}{N}} \\ B_R(R, \frac{\partial R}{\partial X}) &= \left(\frac{P_0}{2M_R \lambda} \right)^{\frac{1}{N}} \left(\frac{1}{K_R - R + R_0} \right)^{\frac{N+1}{N}} \frac{R^{\frac{2N+1}{N}}}{2(3N+1)} \left(\frac{\partial R}{\partial X} \right)^{\frac{1-N}{N}} \\ C_R(R, \frac{\partial R}{\partial X}) &= \left(\frac{P_0}{2M_R \lambda} \right)^{\frac{1}{N}} \left(\frac{1}{K_R - R + R_0} \right)^{\frac{1}{N}} \frac{R^{\frac{N+1}{N}}}{2} \left(\frac{\partial R}{\partial X} \right)^{\frac{1}{N}} . \end{aligned}$$

Substitution into the general dilatation equation (9) leads, after some

manipulation to the equation

$$\begin{aligned}
 & \left[\frac{R_0}{R_1} + \left(1 - \frac{R_0}{R_1}\right)\phi \right] \frac{2N+1}{N} (\phi')^{\frac{1-N}{N}} \phi'' + \\
 & \left[3N+1 + \left(\frac{\frac{R_0}{R_1} + \left(1 - \frac{R_0}{R_1}\right)\phi}{\frac{K_R}{R_1} - \left(1 - \frac{R_0}{R_1}\right)\phi} \right) \right] \left[\frac{R_0}{R_1} + \left(1 - \frac{R_0}{R_1}\right)\phi \right] \frac{N+1}{N} \left(1 - \frac{R_0}{R_1}\right) (\phi')^{\frac{N+1}{N}} \\
 & + 2 \left(\frac{R_0}{R_1} \right)^2 \left[\frac{K_R}{R_1} - \left(1 - \frac{R_0}{R_1}\right)\phi \right] \frac{1}{N} \eta \phi' = 0
 \end{aligned} \tag{31}$$

where

$$\eta = \frac{(R_1 - R_0)^{\frac{N-1}{N+1}}}{\frac{2N}{R_0^{\frac{N+1}{N}}}} \left(-\frac{\chi}{t^{\frac{N+1}{N}}} \right) \left(\frac{2M_R \lambda}{P_0} \right)^{\frac{1}{N+1}} \left[\frac{N(3N+1)}{N+1} \right]^{\frac{N}{N+1}}. \tag{32}$$

The important parameters in expulsion from a tube displaying "locking" material characteristics are thus R_0/R_1 , K_R/R_1 and N .

Solving the differential equation for $\phi(\eta)$ as outlined previously, gives an equation of the form (23) in which the coefficients are given by

$$a_{R1} = \left[\frac{R_0}{R_1} + \left(1 - \frac{R_0}{R_1}\right)\phi \right] \frac{2N+1}{N} (\phi')^{\frac{1}{N}}$$

$$a_{R2} = [3N+1 + \frac{\frac{R_0}{R_1} + (1 - \frac{R_0}{R_1})\phi}{\frac{K_R}{R_1} (1 - \frac{R_0}{R_1})\phi}] [\frac{R_0}{R_1} + (1 - \frac{R_0}{R_1})\phi]^{\frac{N+1}{N}} (1 - \frac{R_0}{R_1}) (\phi')^{\frac{1}{N}}$$

$$a_{R3} = 2(\frac{R_0}{R_1})^2 [\frac{K_R}{R_1} - (1 - \frac{R_0}{R_1})\phi]^{\frac{1}{N}} \phi'$$

and a solution of the form (29) in which

$$F_R(\eta) = \int \left(\frac{2(\frac{R_0}{R_1})^2 [\frac{K_R}{R_1} - (1 - \frac{R_0}{R_1})\phi]^{\frac{1}{N}} (\phi')^{\frac{N-1}{N}} \eta}{[R_0/R_1 + (1 - R_0/R_1)\phi]^{\frac{2N+1}{N}}} \right. \\ \left. + \frac{\frac{R_0/R_1 + (1 - R_0/R_1)\phi}{K_R/R_1 - (1 - R_0/R_1)\phi}}{\frac{R_0/R_1}{1 - R_0/R_1} + \phi} \phi' \right) d\eta \quad (33)$$

2.5 Expulsion Rate

The solution of the dilatation equation (20) can be used to determine the pressure distribution from equation (17), which then allows calculation of the velocity distribution from equation (13). Using the general expression

$$\dot{Q} = 2\pi \int_0^R U r dr$$

the volumetric flow rate for the pressure - limited case is found to be

$$\dot{Q}_P = \frac{\pi N}{(3N+1)} \left[\frac{N(3N+1)}{(N+1)} \right]^{\frac{1}{N+1}} \left(\frac{2R_0\lambda}{K_P M_P} \right)^{\frac{1}{N+1}} e^{-\frac{M_P}{NR_0}(R_1-R_0)\phi} \frac{(R_1-R_0)^{\frac{2}{N+1}} R^{\frac{3N+1}{N}}}{R_0^{\frac{2N+1}{N(N+1)}} t^{\frac{1}{N}}} (\phi_P')^{\frac{1}{N}}. \quad (34)$$

It follows from this that $\phi_P'(0)$ is a convenient measure of the expulsion rate from the tube and can be written explicitly as

$$\begin{aligned} \phi_P'(0) &= (\dot{Q}_P)_{X=0}^N \left(\frac{3N+1}{\pi N} \right)^N \left[\frac{N+1}{N(3N+1)} \right]^{\frac{N}{N+1}} \left(\frac{K_P M_P}{2R_0\lambda} \right)^{\frac{N}{N+1}} \\ &\quad e^{\frac{M_P}{R_0}(R_1-R_0)\phi} \frac{R_0^{\frac{2N+1}{N+1}} t^{\frac{N}{N+1}}}{R_0^{3N+1} (R_1-R_0)^{\frac{2N+1}{N+1}}} \\ &= \frac{1}{3N+2} \left[\frac{1 - (R_0/R_1)^{3N+2}}{(1-R_0/R_1)(R_0/R_1)^{3N+1}} \right] \frac{1}{\int_0^\infty \exp[-F_P(\eta)] d\eta} \end{aligned} \quad (35)$$

where $F_P(\eta)$ is given by equation (27).

The corresponding expressions for the radius-limited case are

$$\dot{Q}_R = \frac{\pi N}{(3N+1)} \left[\frac{N(3N+1)}{(N+1)} \right]^{\frac{1}{N+1}} \left(\frac{2M_R\lambda}{P_0} \right) \left[\frac{1}{K_R - (R_1-R_0)} \right]^{\frac{1}{N}} \frac{(R_1-R_0)^{\frac{2}{N+1}} R^{\frac{3N+1}{N}}}{R_0^{\frac{2}{N+1}} t^{\frac{1}{N+1}}} (\phi_R')^{\frac{1}{N}}$$

and

$$\begin{aligned}
\phi_R'(0) &= (\dot{Q}_R)_{X=0}^N \left(\frac{3N+1}{\pi N}\right)^N \left[\frac{N+1}{N(3N+1)}\right]^{\frac{N}{N+1}} \left(\frac{P_0}{2M_R \lambda}\right)^{\frac{N}{N+1}} \\
&\quad [K_R - (R_1 - R_0)\phi] \frac{R_0^{\frac{2N}{N+1}} t^{\frac{N}{N+1}}}{R_0^{3N+1} (R_1 - R_0)^{2N/N+1}} \\
&= \frac{1}{3N+2} \left[\frac{1 - (R_0/R_1)^{3N+2}}{(1 - R_0/R_1)(R_0/R_1)^{3N+1}} \right] \frac{1}{\int_0^\infty \exp[-F_R(\eta)] d\eta} \quad (36)
\end{aligned}$$

where $F_R(\eta)$ is given by equation (33).

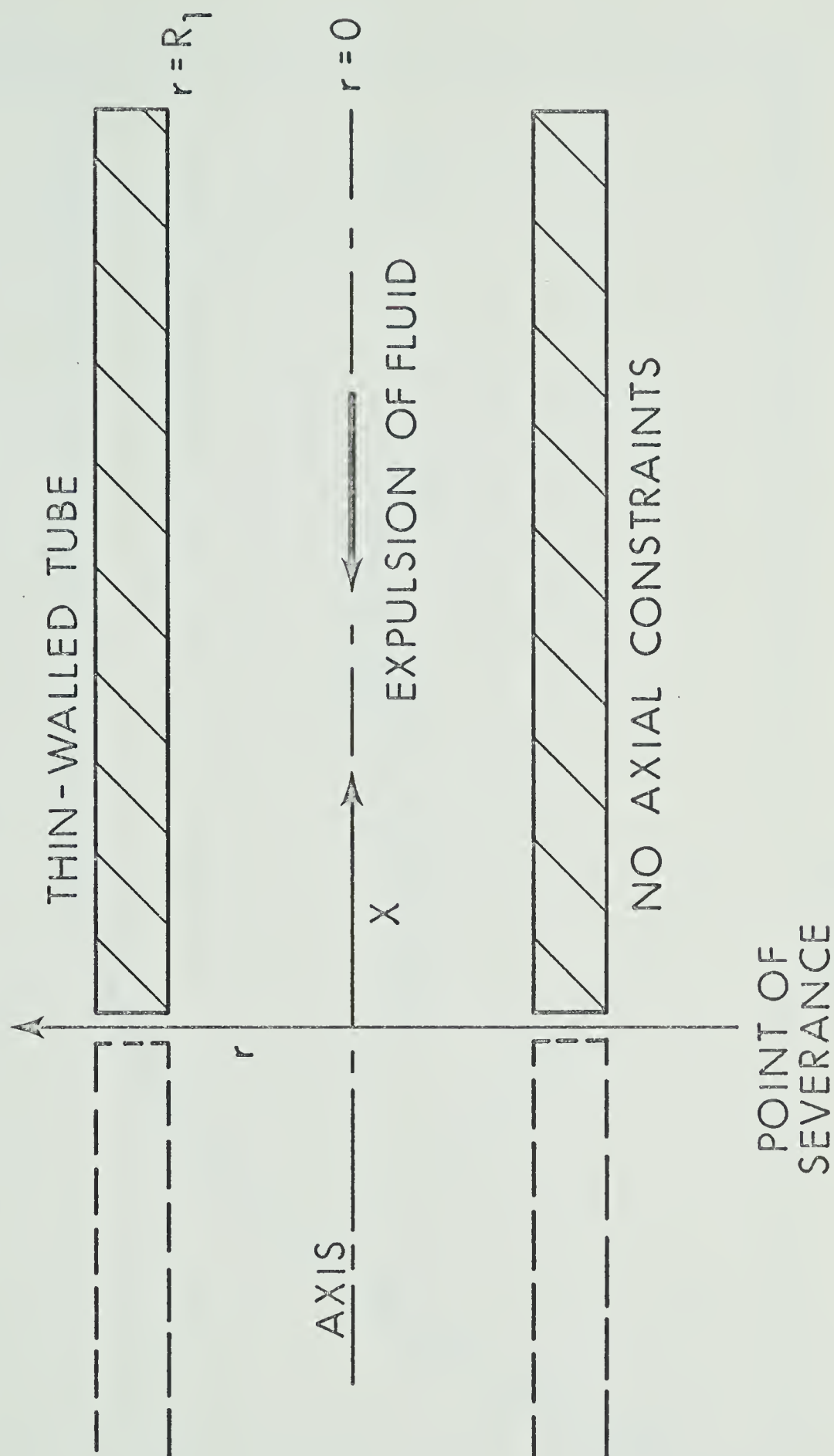
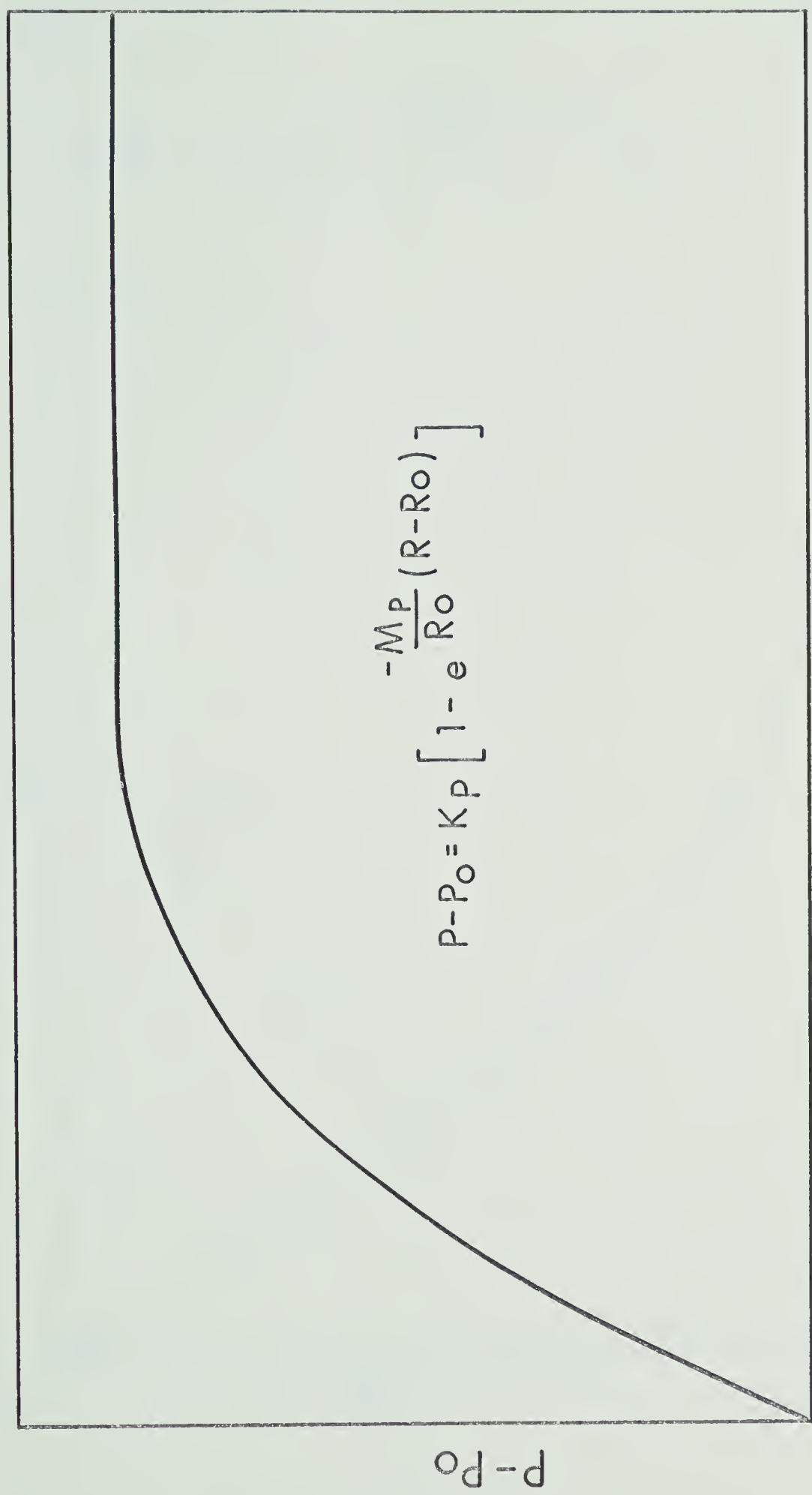


FIG. 2.1 COORDINATE SYSTEM



$R - R_0$

FIG. 2.2 SKETCHED CURVE FOR YIELDING MATERIAL

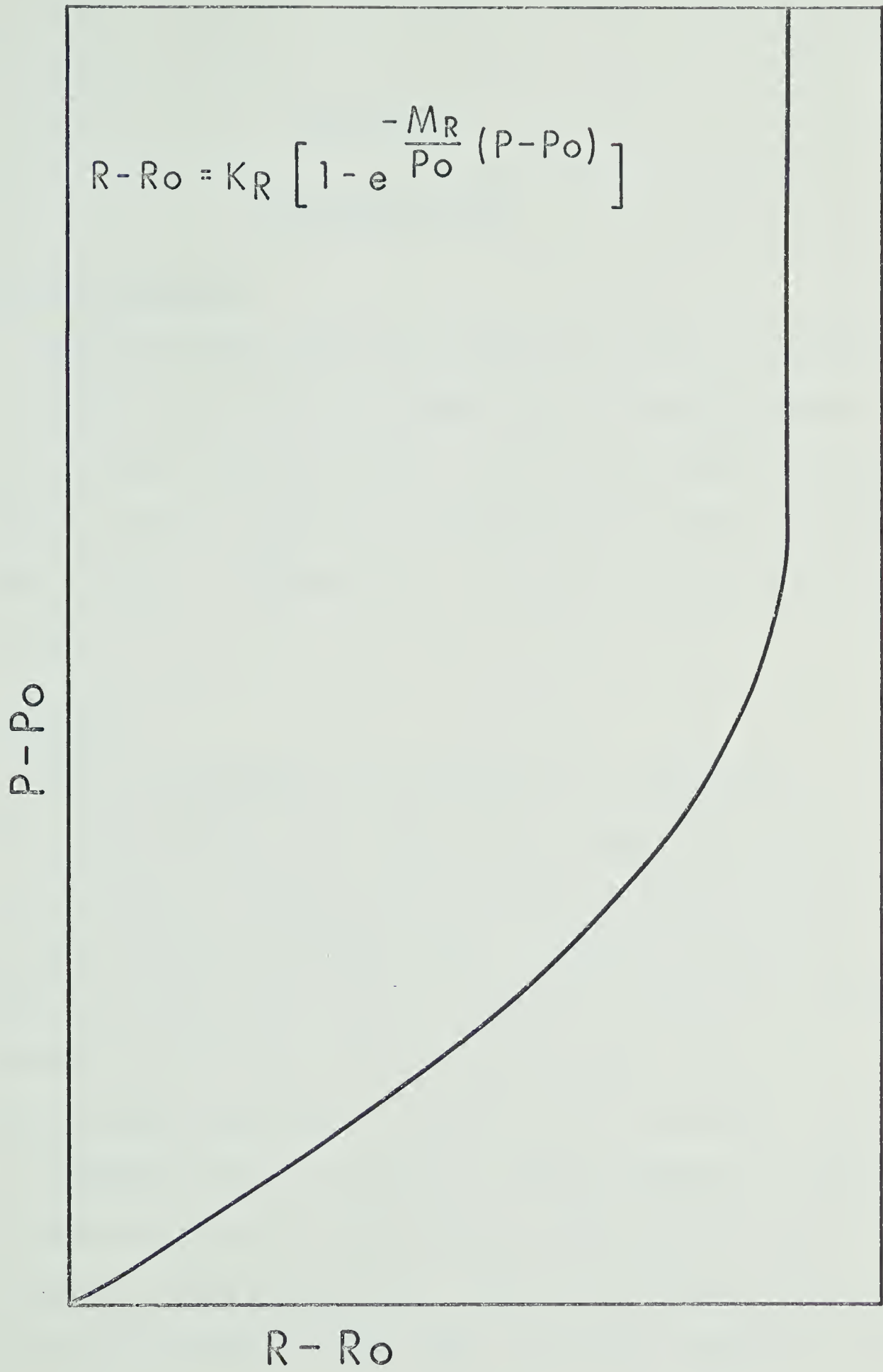


FIG. 2.3 SKETCHED CURVE FOR LOCKING MATERIAL

CHAPTER III

EXPERIMENTAL WORK

3.1 Physical Experiments

To determine the effect on the expulsion process of the wide range of possible values for the parameters R_0/R_1 , N , and M_p would require a very extensive testing program. The experiments here are limited to an important special case, that of a Newtonian fluid ($N=1$) in a latex rubber tube with $M_p = 3.1$ (Appendix A). Two values of R_0/R_1 were used, 0.79 and 0.92.

3.1.a Apparatus

The basic components of the apparatus are illustrated in Figure 3.1. The galvanized steel header tank was supported within a Dexion tower framework, allowing easy positioning to give the desired pressure and dilatation of the latex tube. A length of 1.27 cm I.D. Tygon tubing joined the outlet valve of the tank to another lower valve: this connected with the latex tube in the test section. These Blue Latex Penrose tubes, manufactured in lengths of 91.5 centimeters, were cut to the dimensions shown in Figure 3.2 in preparation for the tests. A detailed description of the latex tubes and their pressure-radius characteristics is given in Appendix A. The latex tubes were purchased from The J.F. Hartz Company Limited, 10606 - 124 Street, Edmonton, Alberta.

Support for the latex tubes was provided by placing them inside an approximately 7 centimeter shorter length of heavy-walled 1.52 cm I.D. Tygon tubing fixed along the "V" portion of the Dexion section. Figure 3.3 shows the trio of inner latex tube, outer Tygon tube casing and Dexion supported by three wires slung between two upper railings of a supporting framework of four metal tubes, each attached at its end to a corner of the rectangular metal end plate.

As the theory assumes the tube to be unrestrained axially, the ends of the outer Tygon tube were covered with thin rubber sheets in which were cut holes to allow for free passage of the latex tube.[#] These rubber sheets prevented a lubricant, in this case water, from flowing out of the Tygon tube, yet fitted loosely enough that the inner latex tube was not constrained. In this way the latex tube could be supported on a water surface thus keeping the axial restraint to a minimum. These features can be seen in Figures 3.3 and 3.4.

In order to sever the latex tube quickly and reproducibly, a spring loaded guillotine was constructed and fitted with adjustable blade mountings to give the steel razor blade its best slicing action. The guillotine is featured in Figures 3.6 and 3.7.

3.1.b Instrumentation

Static pressure inside the latex tube was measured using 2.72 cm., 1.47 mm. I.D. Bardic-Deseret Angiocaths purchased from The Stevens Alberta Co. Ltd., 6912 Farrell Rd., S.E., Calgary, Alberta. Figure 3.5 shows the catheter assembly (right) and its two basic components, a

[#]Balloons serve the purpose very well.

a hollow steel needle (left) which fits into and protrudes slightly beyond a flexible plastic catheter (center). Although designed for medical use they were very suitable here. The assembly was inserted through the wall of the latex tube - penetration being facilitated by the sharp, beveled point on the steel catheter which was subsequently removed leaving the flexible portion protruding slightly into the lumen. The application of a very minute amount of Goodyear Pliobond Adhesive to the probe-tube junction prevented leakage of fluid without noticeably reinforcing the latex tube.

Using a three-way valve to connect the pressure probe and transducer provided a means of flushing clear air bubbles trapped in the measuring system.

Transducer output was fed into a strip-chart recorder to obtain a record of pressure vs time. Details of both transducer and recorder are given in Appendix C.

The recorder's channel-two preamplifier was utilized to record the critical point on the time scale, $t = 0$. One wire from the preamplifier was attached to the quillotine, the other was positioned level with the bottom of the latex tube (Figure 3.4). In this way, the cross-bar of the blade mounting could strike it to complete the circuit and register the event on the strip chart.

Before each series of tests, the recording system was balanced and the transducer calibrated using a mercury manometer.

3.1.c Procedure

With the catheter cemented in place, the latex tube was positioned inside the heavy Tygon tube in such a way that the catheter could be lifted up through the 5 x 0.6 cm. slot in the heavy wall. Once the catheter and transducer had been connected via a three-way valve, the upstream portion of the latex tube was attached to the valve on the tygon tubing leading from the header tank. Before opening this lower valve, the transducer-catheter system was flushed several times, a 20 cc. syringe of the appropriate fluid being applied alternately from opposite ends of the system.

Next, the lower valve was opened and the glycerine-water solution allowed to flow freely through the latex tube. The downstream end of the latex tube was then slowly closed off while the transducer was flushed once more to force any remaining air bubbles into the latex tube and out of the system. Finally, with the distal end tightly clamped, the proximal end, near the lower valve, was also tightly clamped and water was added to the outer tube so that the latex tube would rest on a layer of this lubricating fluid.

At this time, the point of severance, $X = 0$, lay 3.8 cm. beyond the rubber sheet covering the outer tubing. The remaining 7.6 cm. segment of the latex tube was supported near its clamped end by a rubber sling attached to the supporting framework (Figures 3.3 and 3.4).

Finally, after last-second checks, the guillotine was loaded, the recorder activated, and if all systems remained stable, the blade

released and the latex tube severed.

3.1.d Testing Program

Only Blue Latex Penrose tubes were used during the experiments, a new tube being prepared for each test.

With the system filled with a glycerine-water solution of the chosen viscosity, large dilatation tests were performed at locations of 12.7, 38.1 and 63.5 cm from the point of severance. These tests were then repeated with the tank in a lower position to provide small dilatation results. The system was then thoroughly cleaned and refilled with another glycerine-water solution of different viscosity and the procedure repeated. In all, three viscosities were used, 1.26 poise, 4.26 poise, and 10.80 poise. The measurement of viscosity is discussed in Appendix D.

Two additional experiments, VIII and IX, were run at 6.68 poise with an increased length of tubing upstream from the probe (Figure 3.2) to show the effect of making the tube length more closely approach that of a semi-infinite tube as assumed in the theory.

A representative example of the pressure recordings obtained are displayed in Appendix F.

3.1.e Associated Difficulties

No major difficulties were encountered in conducting the tests. Air bubbles remaining trapped in the transducer did pose some problems, the effects of which are shown in Figures 4.7b and F.8. The transducer design prevented detection of air bubbles until flushing at completion

of the test. As glycerine solutions readily trap air bubbles, this problem can only be solved by using a transducer with a clear chamber such as manufactured by Statham, or by exercising extreme care in pouring the solutions and thoroughly flushing the catheter-transducer system.

3.2 Physiological Experiments

The canine venous system was chosen in order to study the expulsion process in a physiological system which could meet the requirements that $\nu t/R_1^2 \gg 1$ and $L/R_1 \gg 1$. Figures 3.8, 3.9, 3.10 and 3.11 illustrate the major veins on which the tests were conducted, namely the cephalic, saphenous and femoral veins. In addition, the larger veins of the mesentery of the small intestine were also used.

3.2.a Instrumentation

The pressure probes used were of the Angiocath type described in section 3.1.b and pictured in Figure 3.5, and had an I.D. of 0.6 mm. The method of insertion was the same as that for the latex tubes except that much greater care and precision of movement was required, the vein diameters being only 3 mm. at most. Catheters were fixed in place with a very minute amount of the experimental Ethicon tissue adhesive IBC - 2, an isobutyl 2-cyanoacrylate monomer.

A 15 cm. length of vinyl tubing I.D. 2 mm. was used to connect the catheter to the pressure transducer.

Severance was effected using fine scissors, at which time the built-in signal marker on the recorder was activated to give some indication

of zero time.

3.2.b Procedure

The dog was first anesthetized with an injection of Nembutal into either the cephalic vein (Figure 3.11) or the peritoneum. One of the cephalic veins, preferably that used in administering the anesthetic was then dissected out and fitted with a cannula for administration of the anticoagulant heparin and more anesthetic when required. Despite the fact that the animal was heparinized, this fixed cannula required frequent flushing with heparinized saline to maintain patency.

In dissecting out each of the veins to provide an area for catheterization and severance, an attempt was made to leave as much surrounding tissue as possible, despite the implication of axial restraint. Usually, the femoral veins had to be cleared of surrounding tissue, whereas it was possible to leave much tissue around the cephalic and saphenous veins. In one test to note, the tissue was completely untouched between the points of severance and pressure measurement.

Tests were performed on both intact veins, in which case the flow was pulsatile due to respiration, and veins in which the downstream end of the vein was clamped, eliminating flow and pulsating pressures.

Following a procedure similar to that used in the physical experiments, the transducer was prepared as outlined in the manual supplied with each transducer and then the system was flushed with 1-2 cc. of heparinized saline, the strip-chart recorder activated, and the vein severed. Following the pressure drop, the vein was clamped to prevent excessive blood loss.

3.2.c Testing Program

Three 5 kilogram dogs were used for the experiments, the routine being the same for each. The test procedure was then executed on the untouched cephalic vein, (Figure 3.11), followed by a femoral vein in the inguinal region (Figure 3.8), the corresponding saphenous vein (Figures 3.9 and 3.10), the femoral vein on the other side and its saphenous vein, and finally the mesenteric veins.

3.2.d Associated Difficulties

Using the cephalic vein for the initial injection of Nembutal was very difficult because of the small size of the vessel in a 5 kilogram dog. In addition, the vein could not be used later for tests because of hematoma, coagulation and collapse. By using an intraperitoneal injection, as with the third animal, these problems were averted, although several minutes were required for the anesthetic to be effective. This procedure is recommended for animals of this size.

Extreme caution had to be exercised in applying the tissue adhesive otherwise it would spread onto the surrounding tissues and venous surface to form a hard brittle coating. The vein could not then be used for the test, the results showing a rapid decrease in pressure, as one would expect from a non-distensible system.

The system of manually activating the signal marking $t = 0$ was inadequate and inconsistent.

Unsuccessfully, attempts were made to perform static loading tests on vein segments to determine their rheological characteristics.

Consequently, the results outlined in [3] were used in estimating the parameters M_R and K_R (Figure B.2) to obtain a theoretical curve to compare with the experimental data (Figure 4.8).

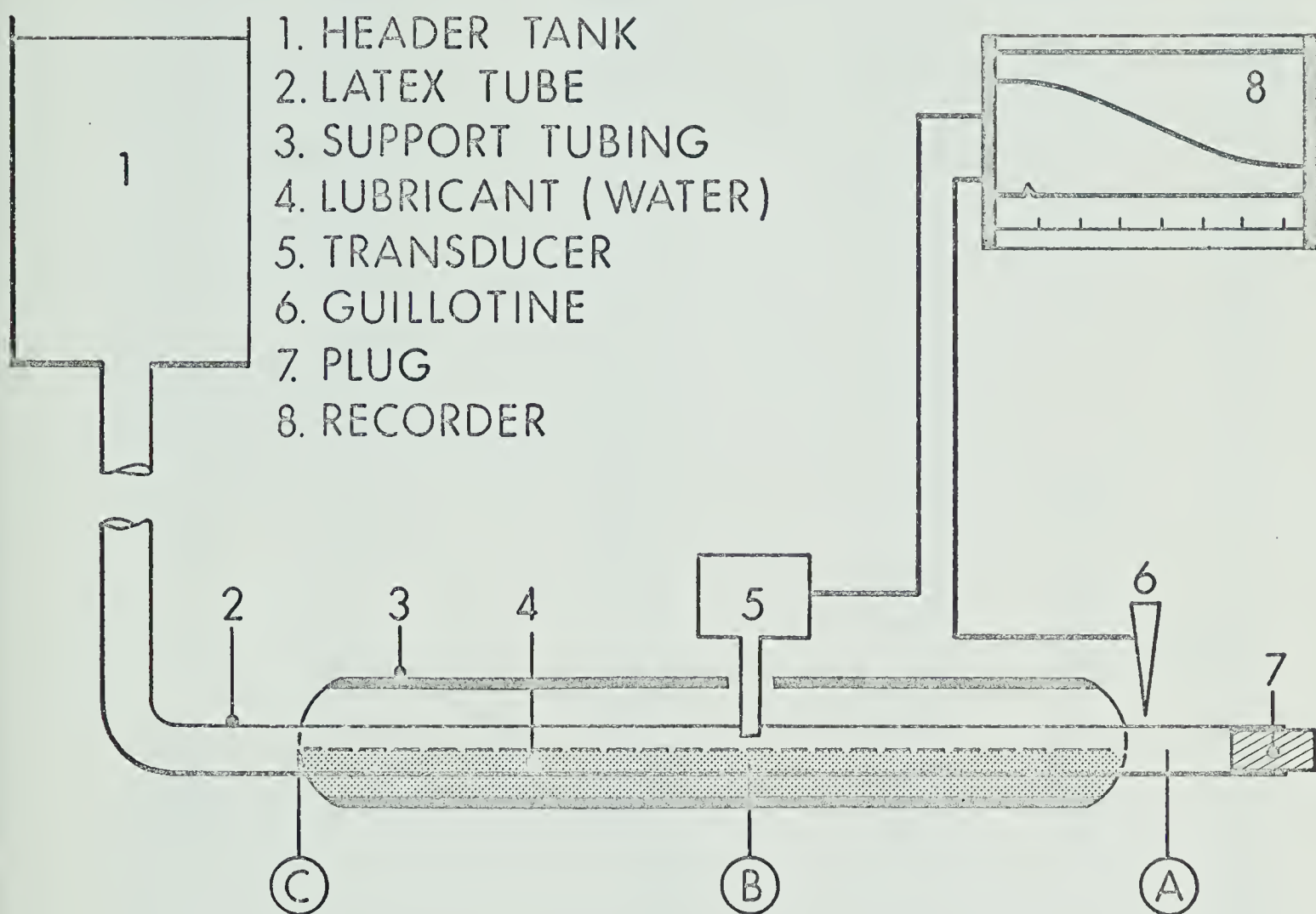
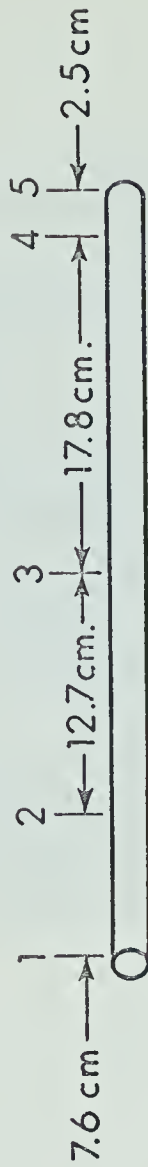


FIG. 3.1 SCHEMATIC OF APPARATUS

TESTS I - VII

1. DOWNSTREAM CLAMP
2. POINT OF SEVERANCE
3. CATHETER
4. UPSTREAM CLAMP
5. ADAPTER



TESTS VIII - IX



FIG. 3.2 TUBING LAYOUTS

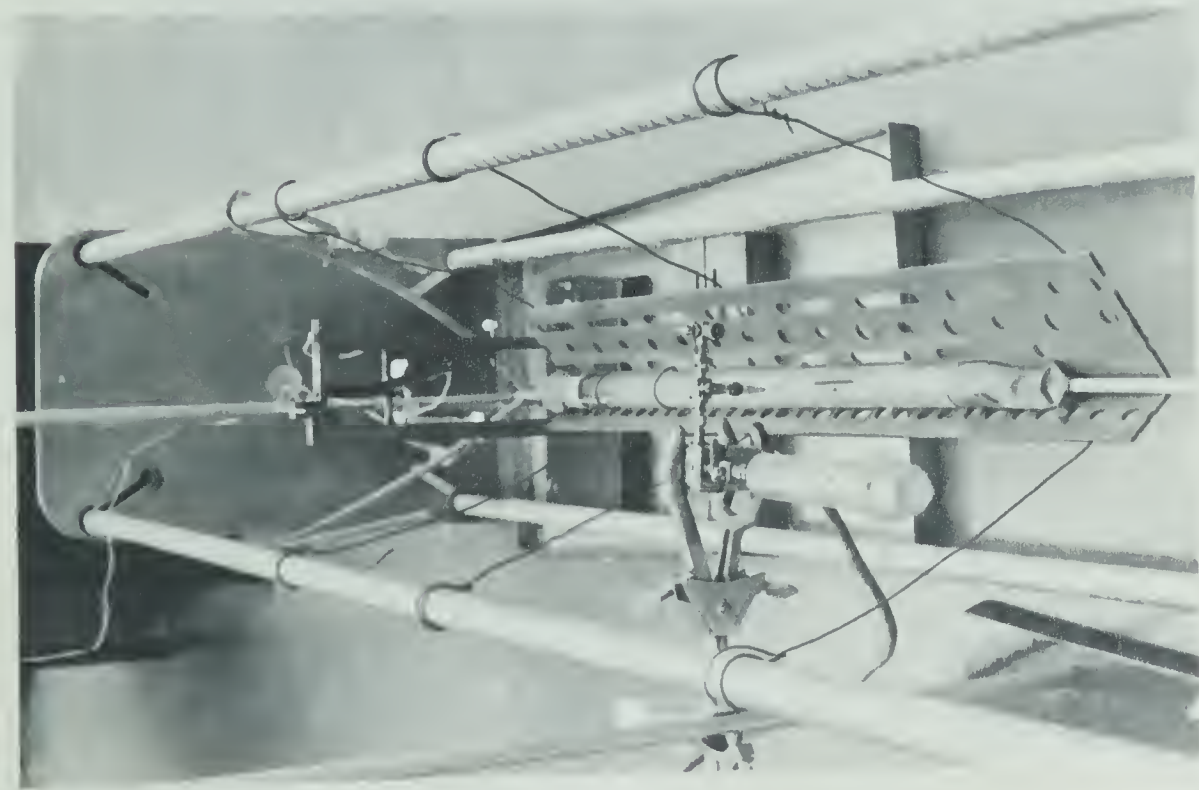


FIG. 3.3 EXPERIMENTAL APPARATUS

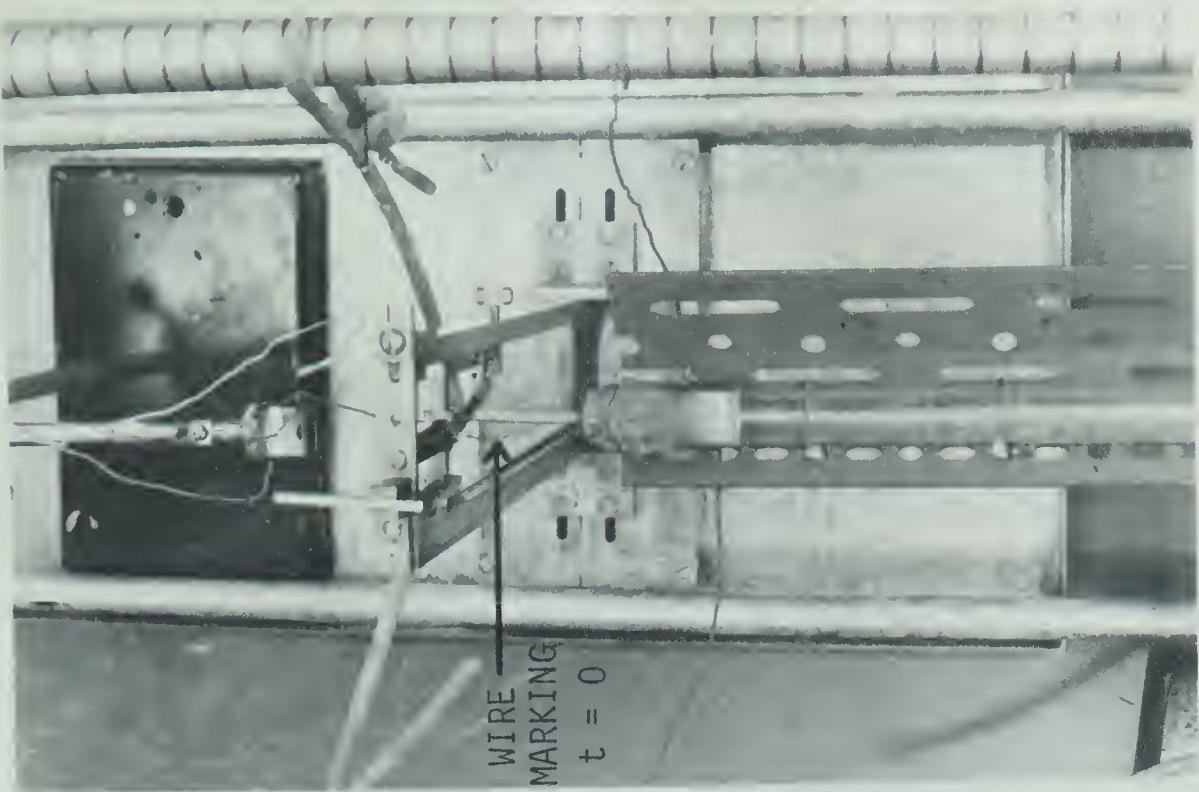


FIG. 3.4 APPARATUS-DETAIL AT
POINT OF SEVERENCE



FIG. 3.5 BARDIC DESERET ANGIOCATH

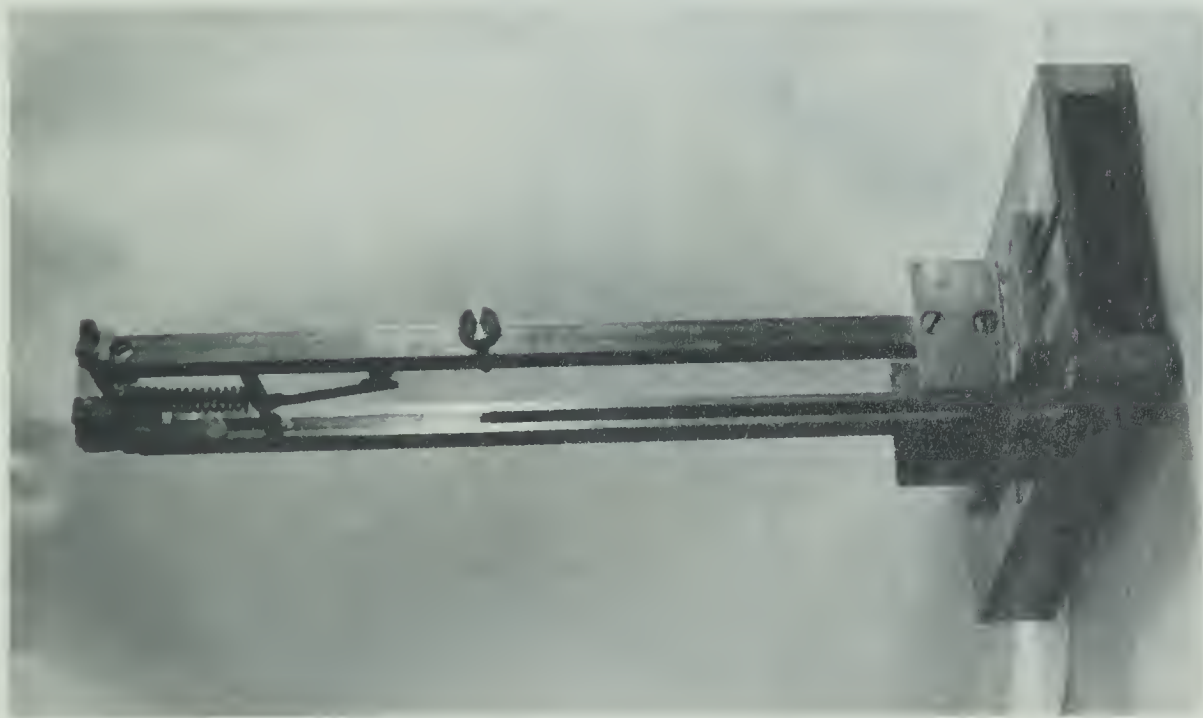


FIG. 3.6 GUILLOTINE-PICTORIAL VIEW

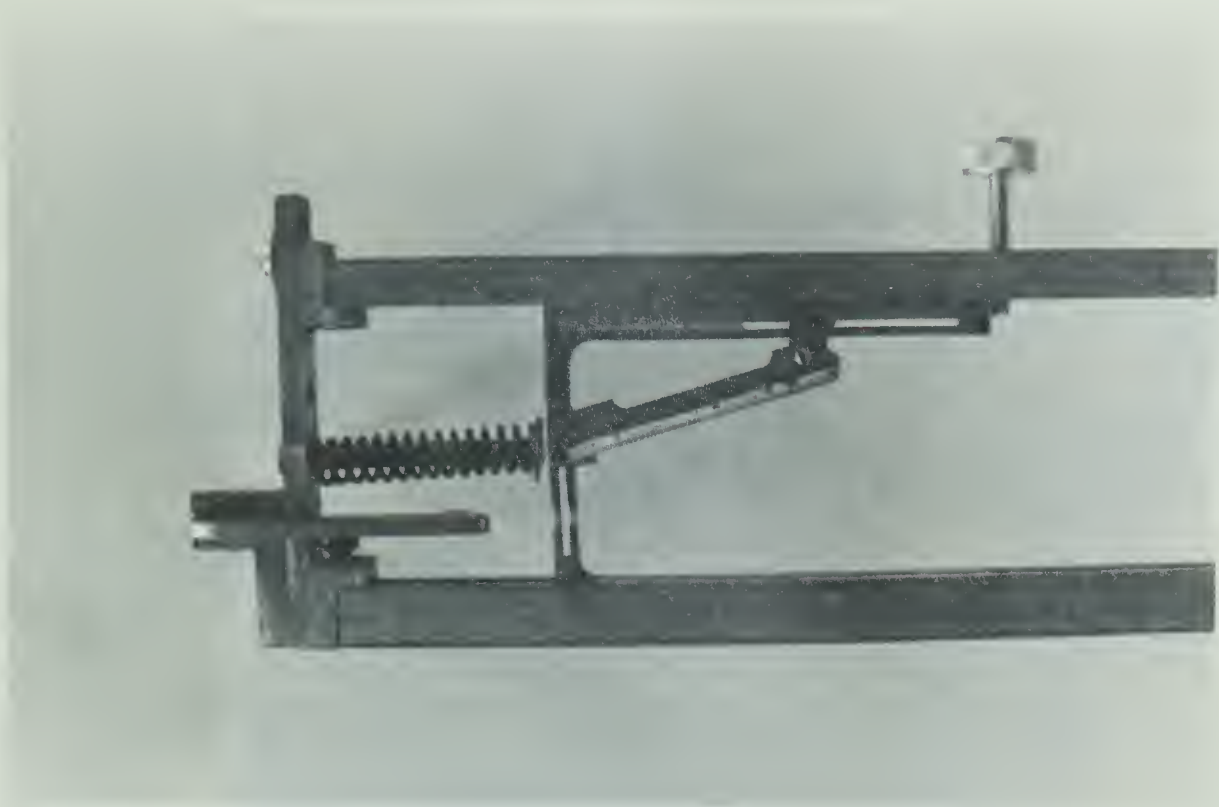


FIG. 3.7 DETAIL OF BLADE AND
SPRING ASSEMBLY

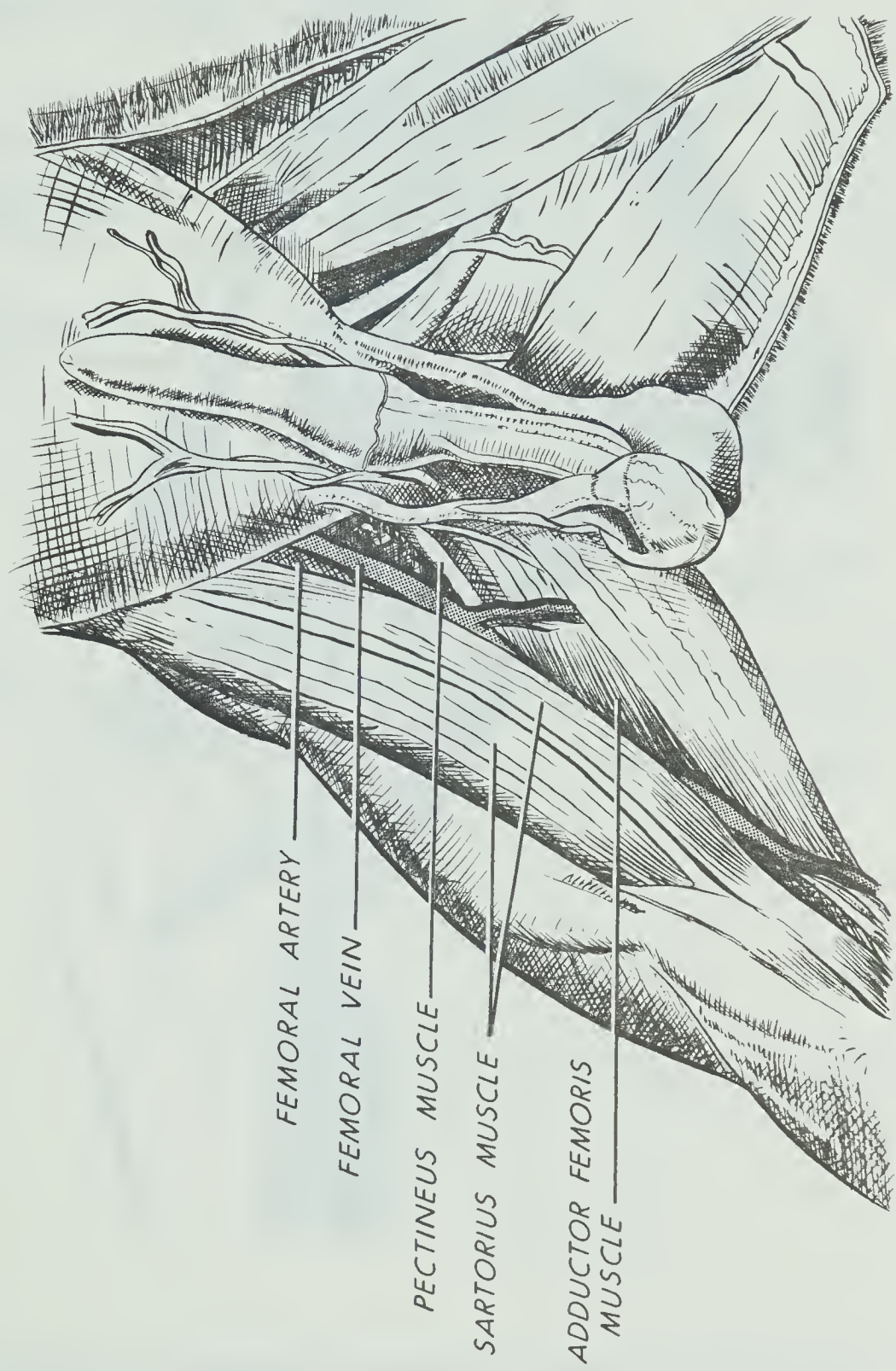


FIG. 3.8 DISSECTION OF INGUINAL REGION AND MEDIAL ASPECT OF THE THIGH

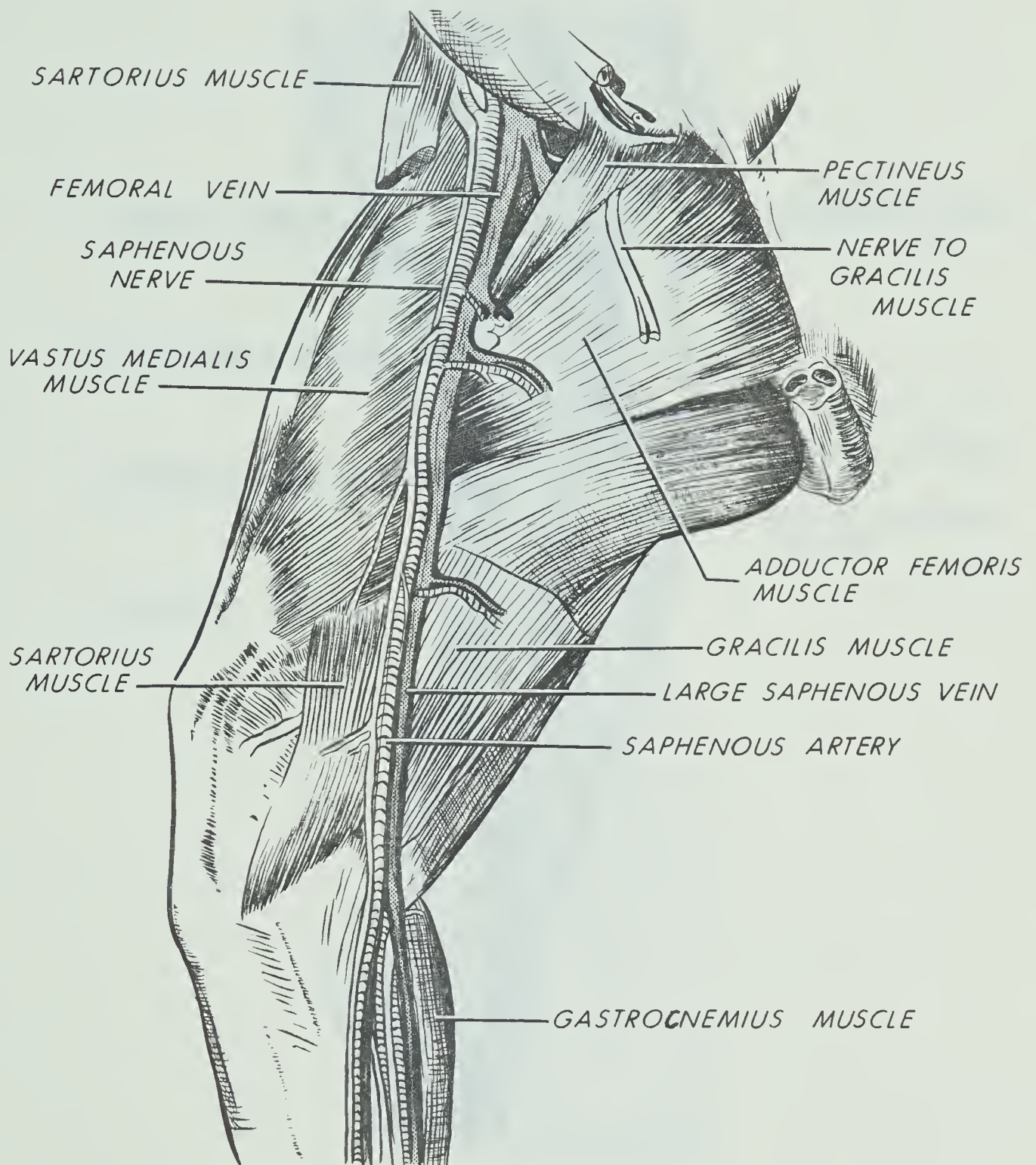


FIG. 3.9 DISSECTION OF MEDIAL ASPECT OF THE THIGH
AFTER REMOVAL OF SARTORIUS AND GRACILIS MUSCLES

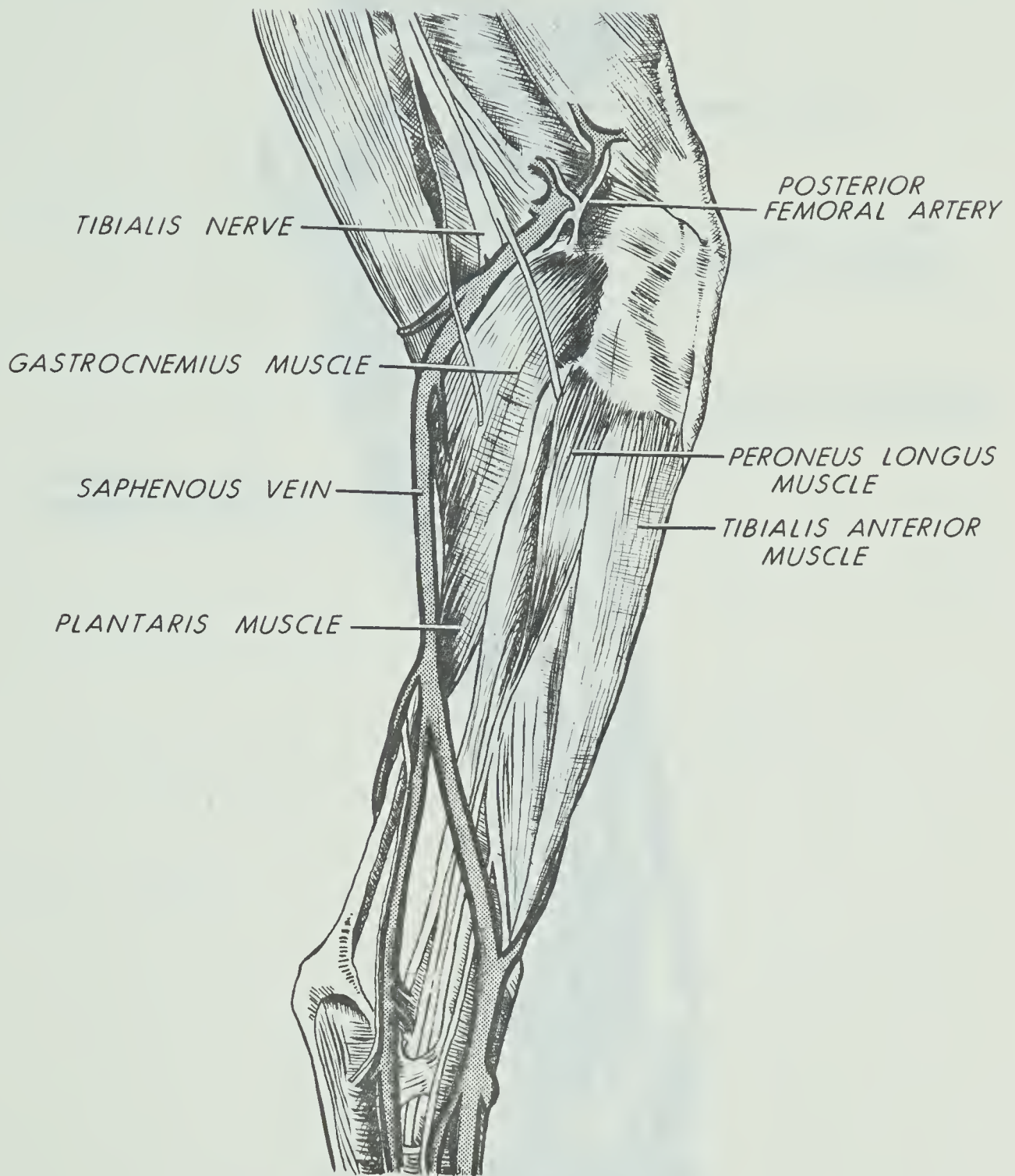


FIG. 3.10 DISSECTION OF LATERAL ASPECT OF THE LEG

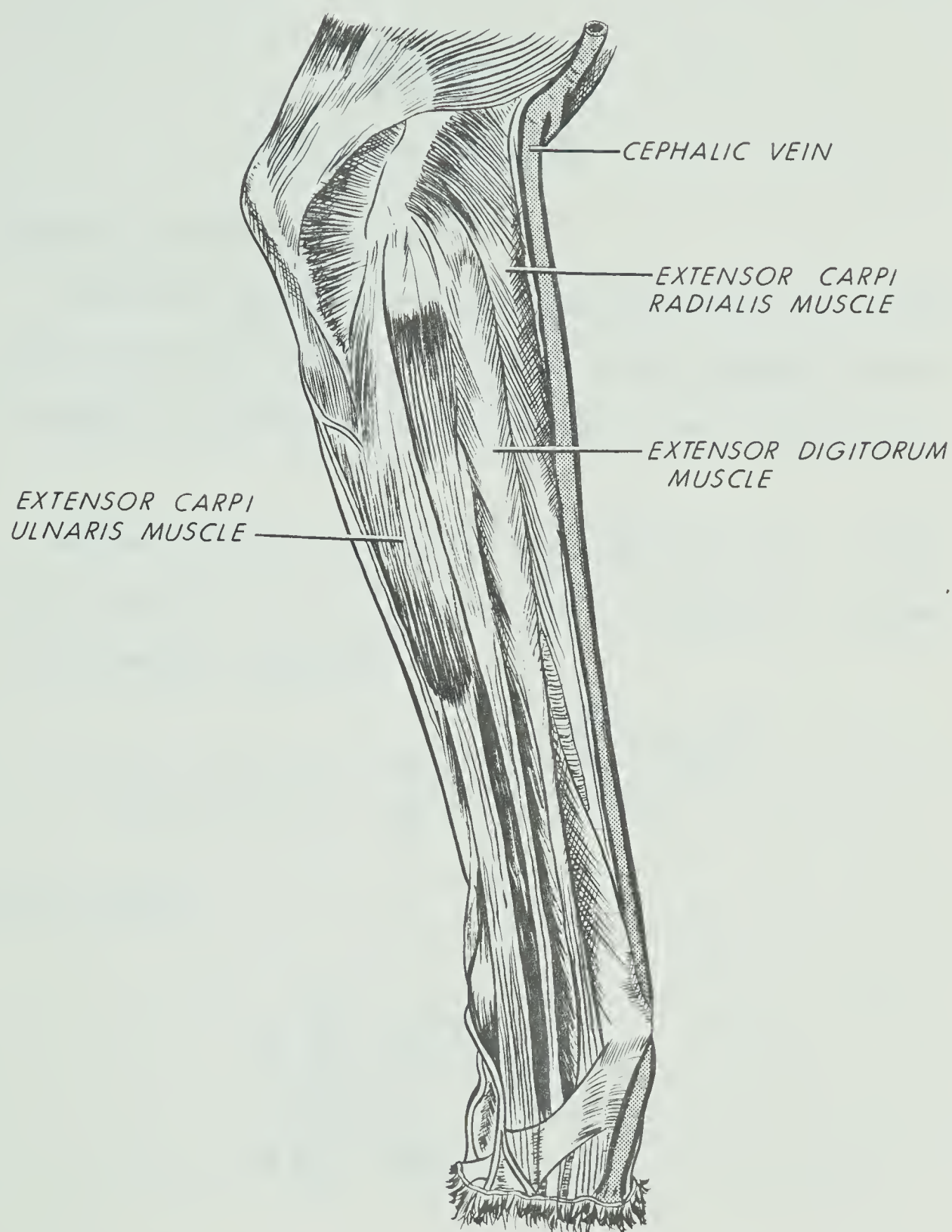


FIG. 3.11 DISSECTION OF LATERAL ASPECT OF THE FOREARM

CHAPTER IV

DISCUSSION OF RESULTS

4.1 Theoretical Dilatation Results

The results presented here deal with the effect of M , N and R_0/R_1 on the dilatation equation. The values of the parameters shown in the accompanying figures were chosen because of their illustrative value.

Consider first the results for a yielding material. If the exponential function for $P - P_0$, equation (17), is expanded in a power series the following is obtained:

$$P - P_0 = K_P - K_P \left[1 - \frac{M_P}{R_0} (R - R_0) + \frac{\left(\frac{M_P}{R_0} (R - R_0)\right)^2}{2!} \dots \right]$$

Choosing M_P so that

$$\frac{\frac{K_P \left(\frac{M_P}{R_0} (R - R_0)\right)^2}{2!}}{K_P \frac{M_P}{R_0} (R - R_0)} \leq 10^{-3}$$

results in $M_P = 0.002$ for $R - R_0 = 0.1$ cm. with a corresponding deviation from a linear relation of 0.1%.

The first set of results, Figures 4.1a to 4.1f show the effect of M_p and R_0/R_1 on the dilatation equation for a Newtonian fluid, $N = 1$. As expected, the results in Figure 4.1a are identical with those displayed by Lock within the tolerance indicated. For any one value of M_p , as the ratio R_0/R_1 decreases, the slope of the tube in the immediate vicinity of the point of severance progressively increases. As Lock [1] pointed out, the analysis is less accurate in this region because of the neglect of bending stresses in the tube wall, Kuchar and Ostrach [4].

When studying the illustrations displaying the effect of the parameters R_0/R_1 , M and N on expulsion rate it should be kept in mind that only eight points were used for any one line. The effect of R_0/R_1 on the expulsion rate is most marked for small M_p (Figure 4.2). As M_p increases, the marked effect of the smaller dilatation ratios on the expulsion rate is less pronounced. Considering the equation for expulsion rate (35) it can be seen that as R_0/R_1 approaches 0, $\phi'(0)$ approaches ∞ , and that this effect is damped by increasing the value of M_p . For $R_0/R_1 = 1.0$ the second term of the dilatation equation (22) drops out and carries with it the influence of M_p . As well, the influence of M_p is removed from the third term. Thus the remaining equation is independent of M_p , a fact which is well illustrated by the convergence present in Figure 4.2 and the invariable position of the graph for $R_0/R_1 = 1.0$ in Figures 4.1a to 4.1f.

The second set of results considers the effect of fluid rheology and dilatation ratio. Looking at the first part of the second term of the

dilatation equation (22) along with the third term it can be seen that a decrease in N would have the same effect as an increase in M_p . This fact is illustrated in Figures 4.3a to 4.3e and 4.4. Again, when $R_0/R_1 = 1.0$ the effect of wall rheology on the dilatation equation is eliminated, but not so the effect of fluid rheology which appears as an exponent in the first and third terms. Thus there is a different value of $\phi'(0)$ for each chosen value of N (Figure 4.4). The dilatation ratio and fluid rheology are related such that for values of R_0/R_1 less than approximately 0.8, an increase in the value of N will cause an increase in the expulsion rate. However, for values of R_0/R_1 greater than approximately 0.8, the opposite condition exists, as N increases, the slope at the origin decreases.

The results for a locking material are displayed in Figures 4.5a to 4.5f. Here, the ratio $1/(K_R/R_1)$ has much the same influence on the expulsion process as did M_p in the case of a yielding material. However, setting $R_0/R_1 = 1.0$ does not remove this parameter from the dilatation equation (31). The position of the line representing $R_0/R_1 = 1.0$ is different for each value of K_R/R_1 (Figures 4.5a to 4.5f), and the curves shown on Figure 4.6 do not converge. For any one value of R_0/R_1 , increasing K_R/R_1 also increases the expulsion rate, the effect being greatest at small R_0/R_1 (Figure 4.6).

4.2 Comparison of Theory and Physical Experiment

The latex tubing chosen for the experiments can be categorized as a yielding material and its behavior well fitted with an exponential

function (Figure A.5).

"Quasi-static" conditions imply that the observed time scale of the expulsion process be greater than R_1^2/ν , which takes on the values 0.15 sec., 0.04 sec. and 0.016 sec. for the three viscosities used. The time scale of the actual expulsion process was only 0.1 seconds for $X = 12.7$ cm. and $\mu = 1.26$ poise which makes these results questionable. However, for all remaining tests the time scale was greater than 0.4 seconds and for $X = 38.1$ cm. or 63.5 cm. and viscosities of 4.26 poise or greater the time scale was several seconds. Tables 1 to 21 give details for each test.

4.2.a Effect of Air Bubbles and Axial Restraint

Experimental results for locations of 38.1 cm. and 63.5 cm. from the point of severance, when considered in relation to the limits of experimental error, satisfied the required conditions of similarity (Figures 4.7a, 4.7c, 4.7d, and 4.7e). This agreement is not shown in Figure 4.7b and was attributed to air bubbles present in the transducer catheter system. Figure G.8 when compared with Figure G.7, reveals that the effect of the more easily compressed air is to act as a "softer spring" and produce a record with a more rapid pressure drop than is actually taking place. Hence, the value of ϕ obtained as shown in Appendix A.4 would be smaller for any particular time or η ; or expressed another way, the recorded time for any one pressure would be smaller than normal and the η variable increased. The end result is the same, the entire curve is shifted to the right.

Imposing an axial constraint on the system has a similar effect as shown by the results in Figure 4.7e and 4.7f, and the pressure records of Figures G.11 and G.12. The tracing of Figure G.11 was taken with sufficient water in the outer tube to allow the inner latex tube maximum freedom of movement. Figure G.12 on the other hand is the record of a dry run, the friction between the tubes preventing the inner tube from sliding freely after severance. The results show clearly that the conditions of no axial restraint assumed in the theory must clearly be met if agreement is to be obtained between theory and experiment.

It should also be pointed out that axial displacements recorded at the point of severance for $X = 63.5$ centimeters and large dilatations were approximately 2.5 centimeters, which are quite significant. Because of the axial restraint, the pressure dropped more rapidly, the effect of which, as described previously, was to shift the respective curves to the right.

Clearly the results obtained for $X = 12.7$ centimeters are not in accord with the theory developed earlier, and cannot be accounted for by experimental error. They will be treated separately at the end of this discussion.

4.2.b Results for $X = 38.1$ and 63.5 cm.

By studying the results displayed for $X = 38.1$ and 63.5 centimeters in Figures 4.7a to 4.7g, and disregarding those results affected by either air bubbles or axial constraints, it can be seen that there is

good agreement between experiment and theory for values of $\eta > 1.0$; that is, for small times. However, for larger times and therefore $\eta < 1.0$ the results consistently fall below the theoretical predictions. If one were to consider a rigid tube filled with fluid under pressure and then instantaneously severed, the resulting pressure curve would drop sharply. In other words, there would not be the effect of the elastic walls closing down and maintaining the pressure. Earlier discussion shows that a pressure curve dropping more rapidly than expected causes a shift to the right when the results are plotted as ϕ vs η . It can thus be conceived that the requirement of semi-infinite tube lengths was not upheld throughout the entire experiments. Close agreement was achieved only to the point when the effect of severance, that is, the decreasing pressure and constriction of the latex tube, reached the upstream clamp. Since this distance was kept constant at 17.8 centimeters for all tests except VIII and IX, the effect should be the same in all tests, as it is, (except for $X = 12.7$ centimeters, for other reasons).

Further proof that behavior of the experimental curves $X = 38.1$ cm. and $X = 63.5$ cm. is due to lacking semi-infinite characteristics is given by Figure 4.7.g. This graph illustrates the results of two separate tests undertaken for $X = 38.1$ centimeters, but with catheter to clamp lengths of 47.0 centimeters, $L/R_1 = 210$ rather than the 17.8 centimeters, $L/R_1 = 140$, used in all previous tests. Results are as expected. There

is good agreement between experiment and theory for a considerably longer period. It could thus be postulated that were it possible to extend this length up to the realm of the semi-infinite, that the agreement would indeed be excellent for all η . However, apart from the fact that the latex tubes were manufactured only in 91.4 centimeter lengths, extending the length of the tube upstream would also increase the amount of axial displacement experienced at the point where the catheter was inserted into the tube for pressure measurements. The flexible nature of the catheter allowed it to cope with elongations which were of approximately 1.8 centimeters magnitude at that point and approximately 2.5 centimeters at the point of severance with a catheter to adapter length of 47.0 centimeters and a dilatation ratio of 0.79. An increased catheter-adapter length would necessitate mounting the catheter-transducer system on a relatively frictionless surface to allow it to follow any axial strain occurring in the latex tube.

In addition, the pressure record shown in Figure G.13 reveals that the time required for the effect of severance to reach a point 38.1 centimeters away is roughly 0.15 seconds. Since $\eta \propto X/\sqrt{t}$, the time required to record a corresponding η a distance of 17.8 centimeters away would be 0.32 seconds which for $X = 38.1$ centimeters corresponds to an η of 0.92. This was in the same region at which all other tests conducted with catheter-adapter lengths of 17.8 centimeters began to fall below the theoretical curve. Now following the same reasoning, increasing the catheter adapter length to 47.0 centimeters would mean that

the effect of severance would be felt at the adapter, 85.1 centimeters from $X = 0$, in 0.75 seconds, which should correspond to an η in the region of 0.5: the figure reveals $\eta \approx 0.6$. It follows that for agreement down to η 's of approximately 0.18, tubes of at least 280 centimeters in length would be required for an $X = 38.1$ centimeters and viscosity of 6.68 poise.

4.2.c Results for $X = 12.7$ cm.

The pattern of results obtained for locations of 12.7 cm. requires additional explanation. That the catheter-to-clamp length was only 17.8 cm. was the dominant force in causing the results to fall consistently below the theoretical curve at $\eta = 0.35$. Figure G.9 shows that the time taken for the effect of severance to reach the catheter at $X = 12.7$ cm. was 0.025 sec. With the procedure used earlier it can be shown that severance effect would reach the clamp, 30.5 cm. from $X = 0$ in 0.038 sec., putting $\eta = 0.36$ which is very close to the observed value of 0.35. As will be discussed later, this effect was not noticed in the physiological experiments where the vein was not clamped upstream and the system could be considered semi-infinite. The experimental curve remained far to the left of the theoretical and did not cross below.

Due to the fact that the theory neglects the effect of bending stresses in the tube wall, it was first believed that the behavior shown could be explained on this basis. However, results obtained by Kuchar and Ostrach [4] show that the elastic entrance length, the region in

which bending stresses must be considered, is given approximately by $Le = 2.67 \sqrt{h R_0}$, where h = tube wall thickness and R_0 = the unstressed tube radius. A quick calculation shows $Le = 2.67 \sqrt{0.0262 \times 0.296} = 0.235$ cm. which is certainly less than 12.7 cm. Thus the explanation must lie elsewhere.

Another explanation appears to lie with the inherent properties of latex rubber tubes. Martin [5] achieved good agreement between theory and experiment when using latex rubber tubes to investigate the linearized dynamic equations of viscous incompressible fluid flow in viscoelastic tubes. The pulsation frequencies used in these experiments were from 0.37 cycles/sec to 4.24 cycles/sec. The time scale for the expulsion process at $X = 12.7$ centimeters and low viscosities would lie within this range. Also, as stated by Harwood and Schallamach [6], viscoelastic losses predominate at short elongation times for natural rubber. This information suggests that the behavior for $X = 12.7$ centimeters may be due to viscoelastic effects. Since the observed time scale for the expulsion process was shortest for $X = 12.7$ centimeters, it seems reasonable to expect that time dependent properties would have more effect on the elastic expulsion process at this particular point, and deviation from the theory greatest. However, the time involved for the pressure to drop from P_1 to P_0 at $X = 63.5$ centimeters using a viscosity of 1.26 poise and the same dilatation ratio, was less than the corresponding time for $X = 12.7$ centimeters and a viscosity of 10.80 poise. If time dependent properties were the cause of the marked

deviation of $X = 12.7$ centimeters results, then they should also have this same effect at $X = 63.5$ centimeters for low viscosities. They did not. Nor were the results for higher viscosities and hence larger times consistently better than those at low viscosities.

Another source of the deviation could stem from the fact that immediately after severance has occurred the tube moves axially in a direction opposite to the flow and regains its initial length. The acceleration of the tube in the direction of higher pressure would also result in higher pressures being recorded and thus yield values of ϕ greater than predicted. This effect would be more noticeable the closer one approached $X = 0$, because the relative axial elongation at this point is the greatest. But once again the explanation is inadequate because of the fact that although the distance X was varied from 12.7 to 63.5 centimeters, the point at which the pressure was measured remained constant relative to the fixed upstream end of the latex tube. Therefore, the elongation and acceleration would be the same for all tests.

Finally, the only remaining possibility is that the condition $\phi(0) = 0$ was not realized instantaneously. It may be that time dependence arose such that the transformation $R - R_0 = \xi(t) \theta(\eta, t)$ would have to be used, $\xi(t)$ replaces $(R_1 - R_0)$ and $\theta(\eta, t)$ replaces $\phi(\eta)$. Although it is highly unlikely that $R_1 - R_0$ is a function of time in this situation, the replacement of $\phi(\eta)$ by $\theta(\eta, t)$ is justifiable and may explain the discrepancy between theory and experiment.

4.3 Comparison of Theory and Physiological Experiment

4.3.a Choosing a Model

The choice of the locking material equations to represent expulsion from veins has good merit. As stated by Burton [15], the peculiar behavior of blood vessels has recently been shown to result from the combination of elastin and collagen fibers in the wall. The elastin fibers are brought into action by a slight stretching of the wall, and the much less extensible collagen fibers do not reach their unstretched length until the vessel is considerably distended. The collagen fibers form a protective limiting "jacket". A similar structure of rubber with a canvas jacket has been used in garden hose. These observations have also been noted by others [3,13,16]. Assuming the fluid to be Newtonian is also acceptable. According to Burton [15], in the physiologic range of blood flow, blood behaves as if it were a Newtonian fluid. Axial accumulation occurs, but reaches a saturation value at a very low velocity of flow.

Veins are copiously supplied with nerves which Thompson [11] demonstrated to be capable of producing constriction of the vein. This innervation was shown to be sympathetic in nature, Donegan [12], and further study by Maynard [13] revealed it to be purely constrictor without any dilator component. In addition to this general constrictor activity there is a more extreme and more substantial form of constriction termed venospasm. There have also been occasional reports of localized rings of constriction, at times so severe as to distort the vein so that it resembles a string of sausages. Such is the case in the mesenteric veins

of the dog, (Alexander [3]).

Alexander [3], also points out that pressures measured in the venous system are highly dependent upon extravascular pressures. The effect of vasoconstriction and pressure from surrounding tissue was well illustrated in one test involving the cephalic vein. The vein was exposed only at two points, one for severing, the other, 5 cm. distal, for pressure measurement. All tissue between the points was left intact. When the vessel was severed, only a small amount of blood exuded from the vessel, and the pressure remained constant. Upon removing the surrounding tissue, the pressure fell.

A more suitable theory than that applied here should incorporate the collapsible nature of the vessels which causes the cross section to be elliptical rather than circular as well as accounting for local venospasm due to venopuncture or severing the vessel.

Thus it does not seem reasonable to expect the same degree of correlation between experiment and theory here as was achieved with the physical system. What could one therefore expect?

4.3.b Interpretation of Results

As much of the surrounding extravascular tissue was unavoidably removed when points of severance were within 2.5 cm. of the point of pressure measurement, the principle phenomena acting when the vein was severed would be venospasm, a general vasoconstriction, and collapsibility. The result would be to maintain the pressure in the vessel despite a reduced volume. Truly this was the case as shown by the pressure recordings

of Figures G.14 and G.15. With viscosities in the range of 0.1 poise, (Haynes [14]) extrapolation of results from the physical experiments would predict an almost vertical descent for the pressure curve, even with the recorder operating at 100 mm/sec. However, because of the active elements present, pressure was maintained and the corresponding values of ϕ were displaced high and to the left of those predicted by theory (Figure 4.8).

Determining the rheological parameters for the veins used posed several problems because of the lack of available data in this field. As outlined in Appendix B the values finally obtained for K_R and M_R as well as the dilatation ratio R_0/R_1 were merely qualitative and thus it was decided to display the results on one figure. The theoretical curve shown was obtained using a dilatation ratio of 0.4, $K_R/R_1 = 0.8$ and $M_R = 0.393$. Assuming other values for the dilatation ratio, Figures I.1 and I.2, did not change the qualitative conclusions.

4.3.c Complicating Factors

The number of possible tests which could be conducted was reduced considerably on account of several factors. First, anesthetizing the animal intravenously prevents the further use of this vein for tests, hence the advantage of intraperitoneal injection. Vasospasm caused by manipulating the vein and inserting the catheter for pressure measurement was at times so severe as to cause complete collapse, preventing any test to be carried out. This occurred in both cephalic and saphenous veins.

The animals withstood the surgery very well, and were still alive after 8 hours, the length of time required to carry out the tests on one animal. They were then administered an overdose of anesthetic.

In summary, the physiological experiments were very difficult to perform, requiring skill and precision acquired only through practice. More extensive testing in a well-equipped laboratory is warranted.

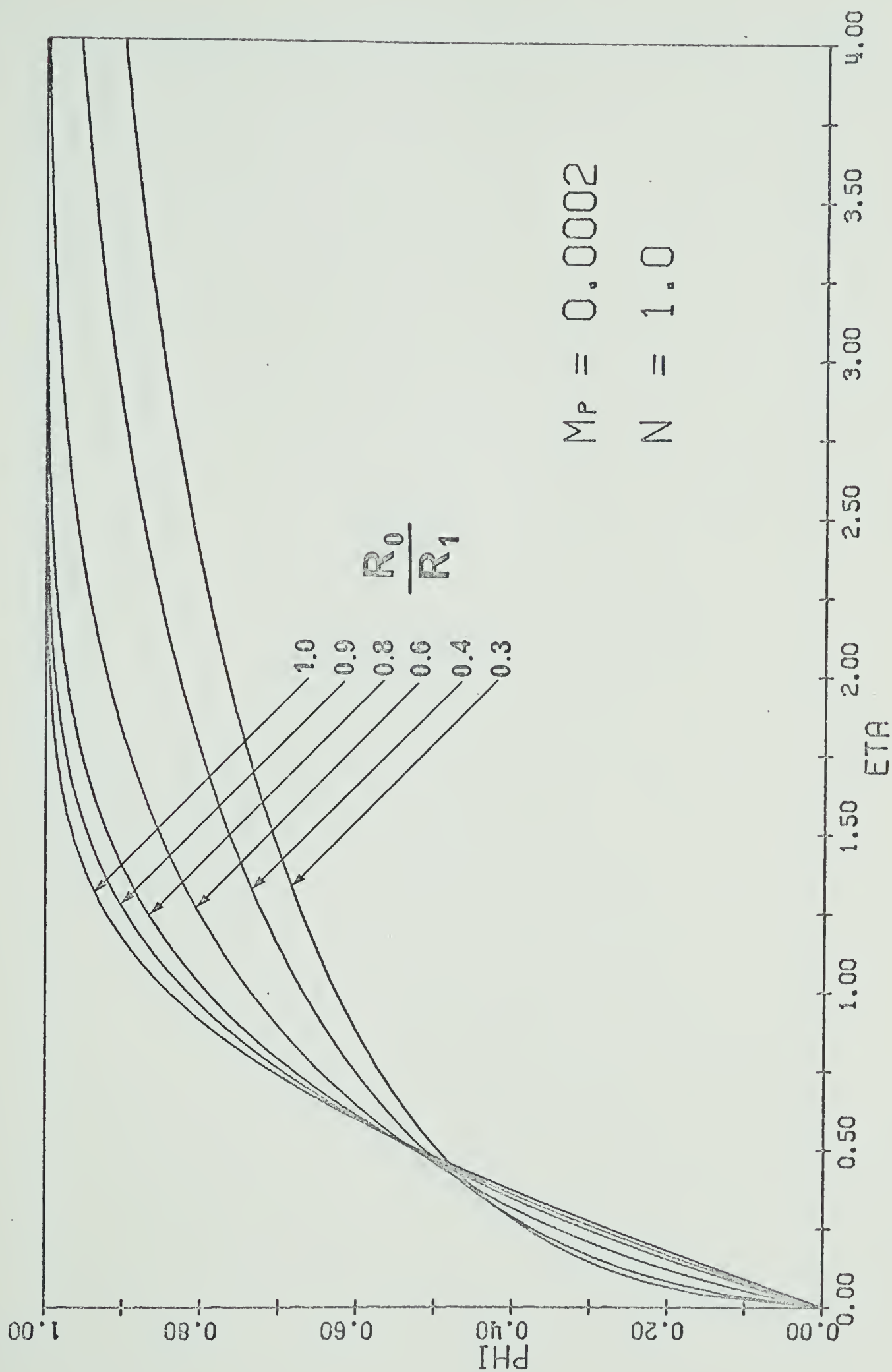


FIG. 4.1.a ϕ VS η , NEWTONIAN FLUID, LINEAR WALL RHEOLOGY

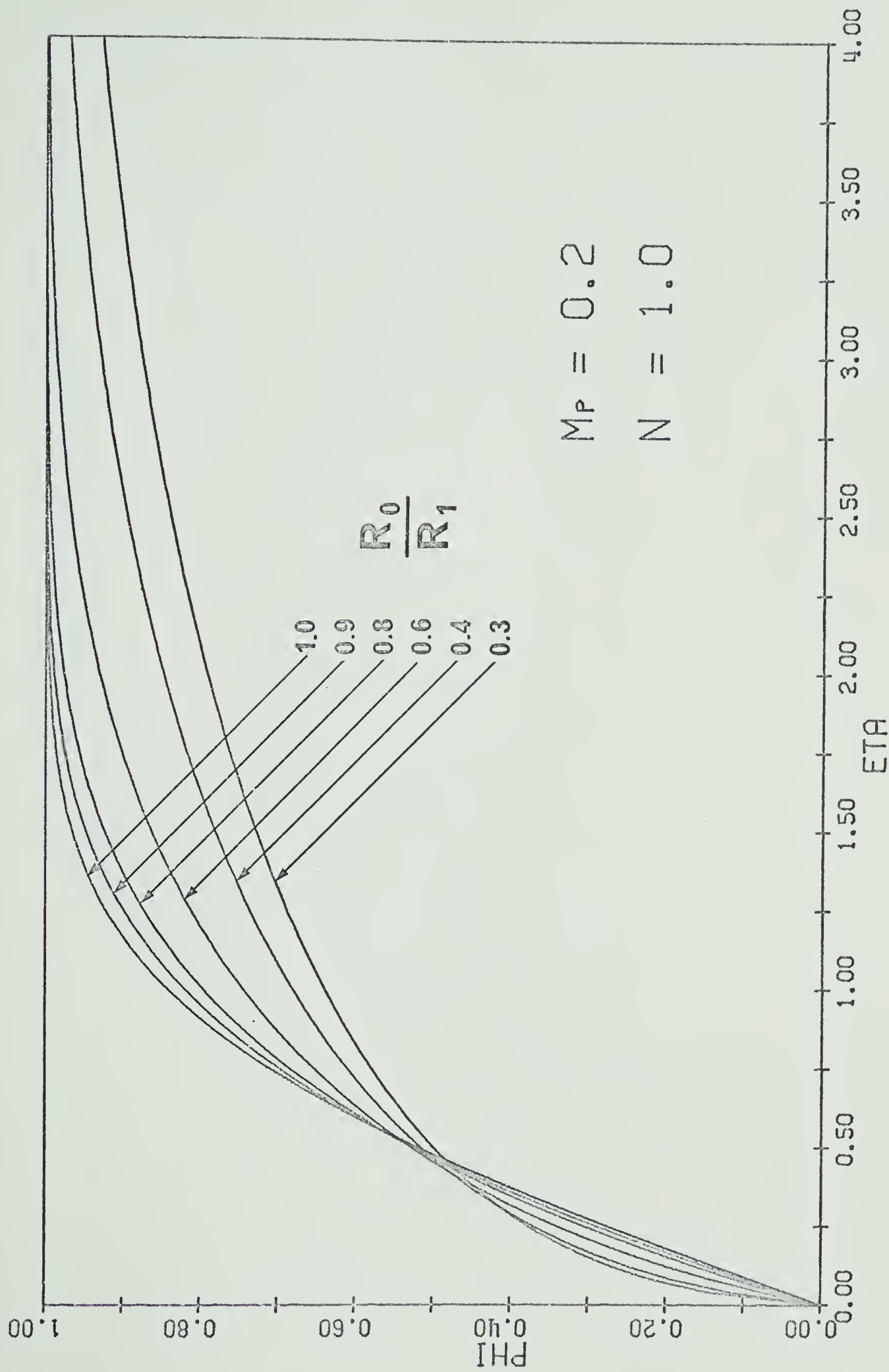


FIG. 4.1.1.b ϕ VS η , NEWTONIAN FLUID, $M_p = 0.2$

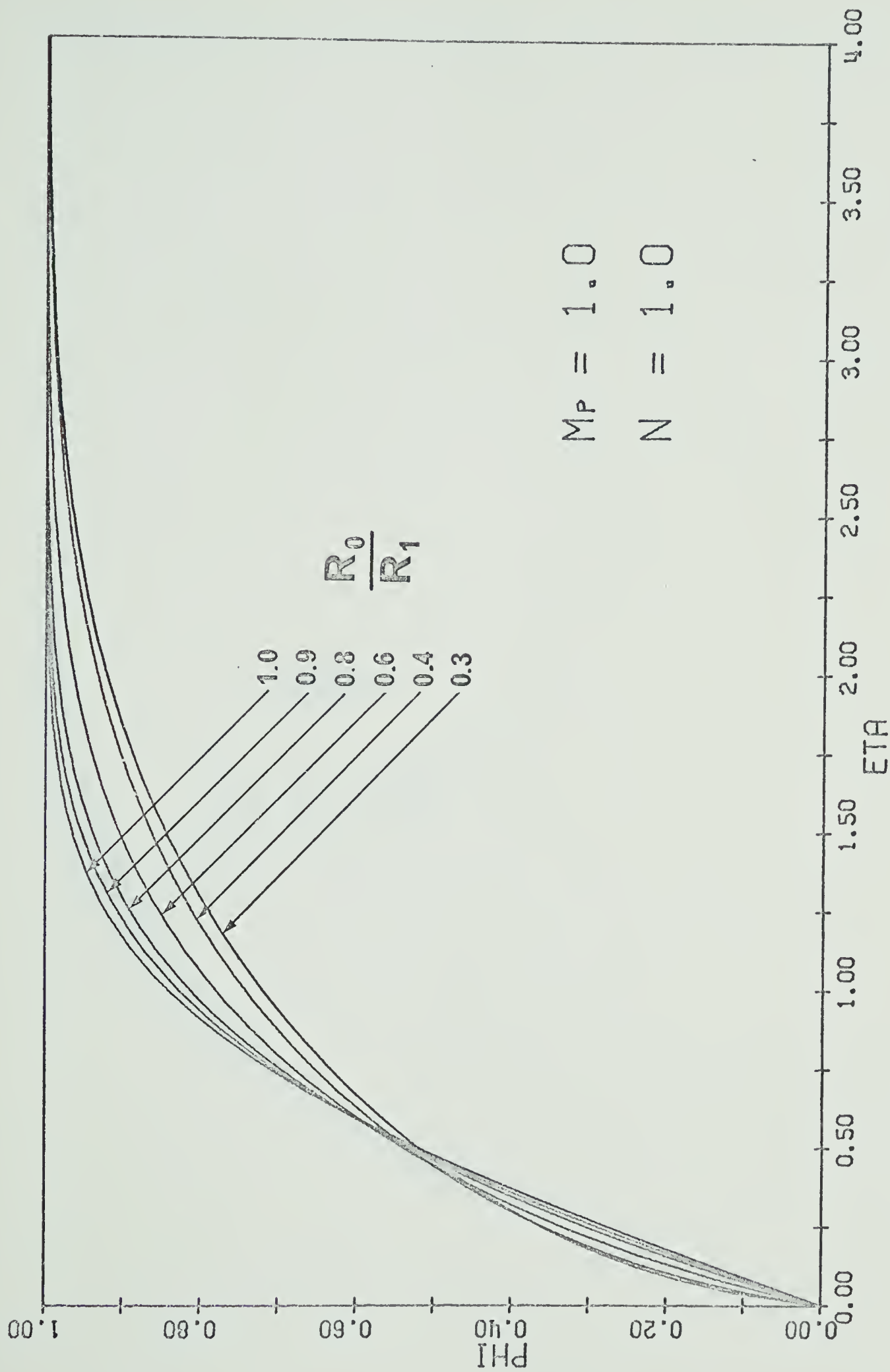


FIG. 4.1.1.c ϕ VS η , NEWTONIAN FLUID, $M_p = 1.0$

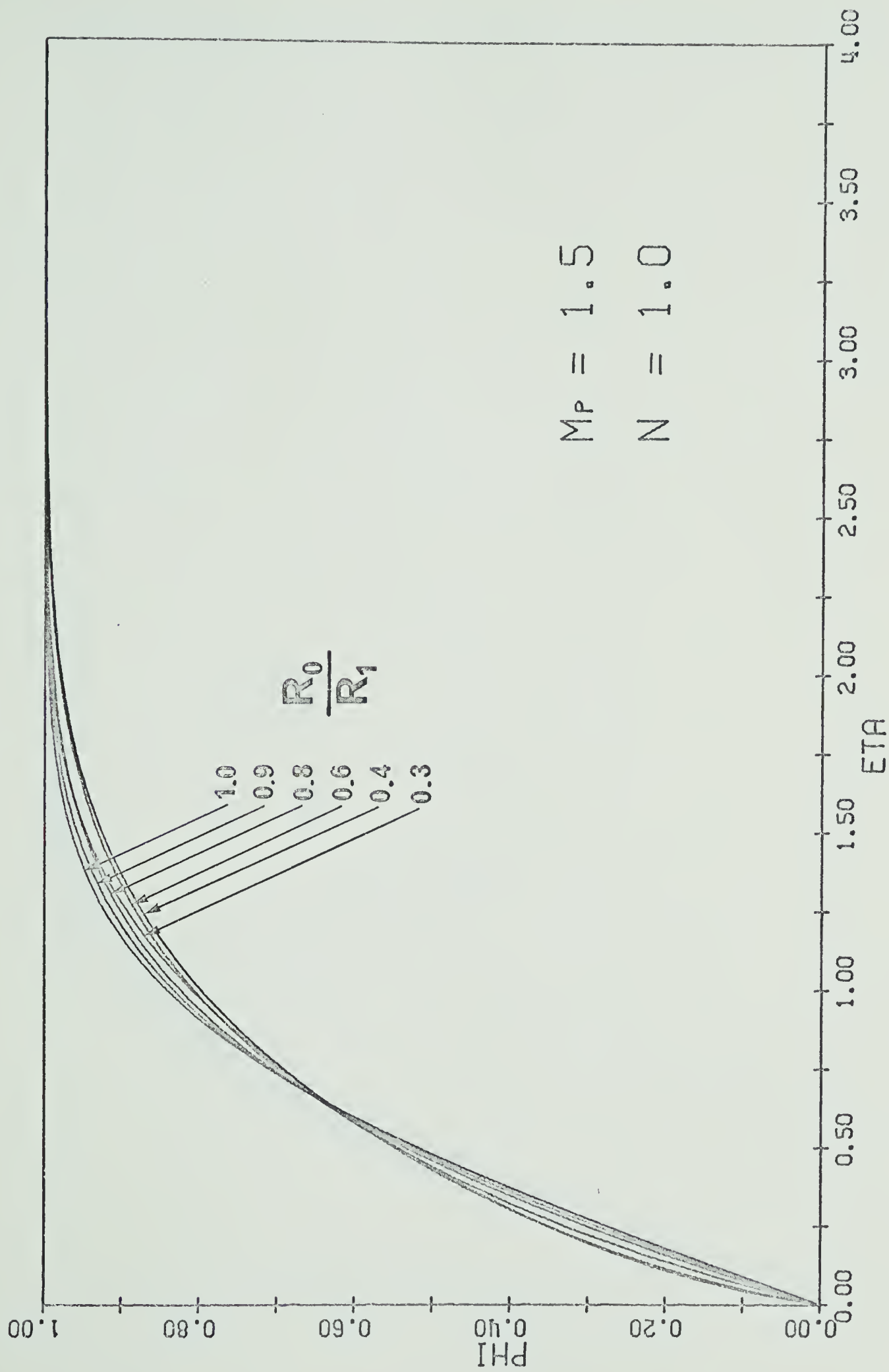


FIG. 4.1.d ϕ VS η , NEWTONIAN FLUID, $M_P = 1.5$

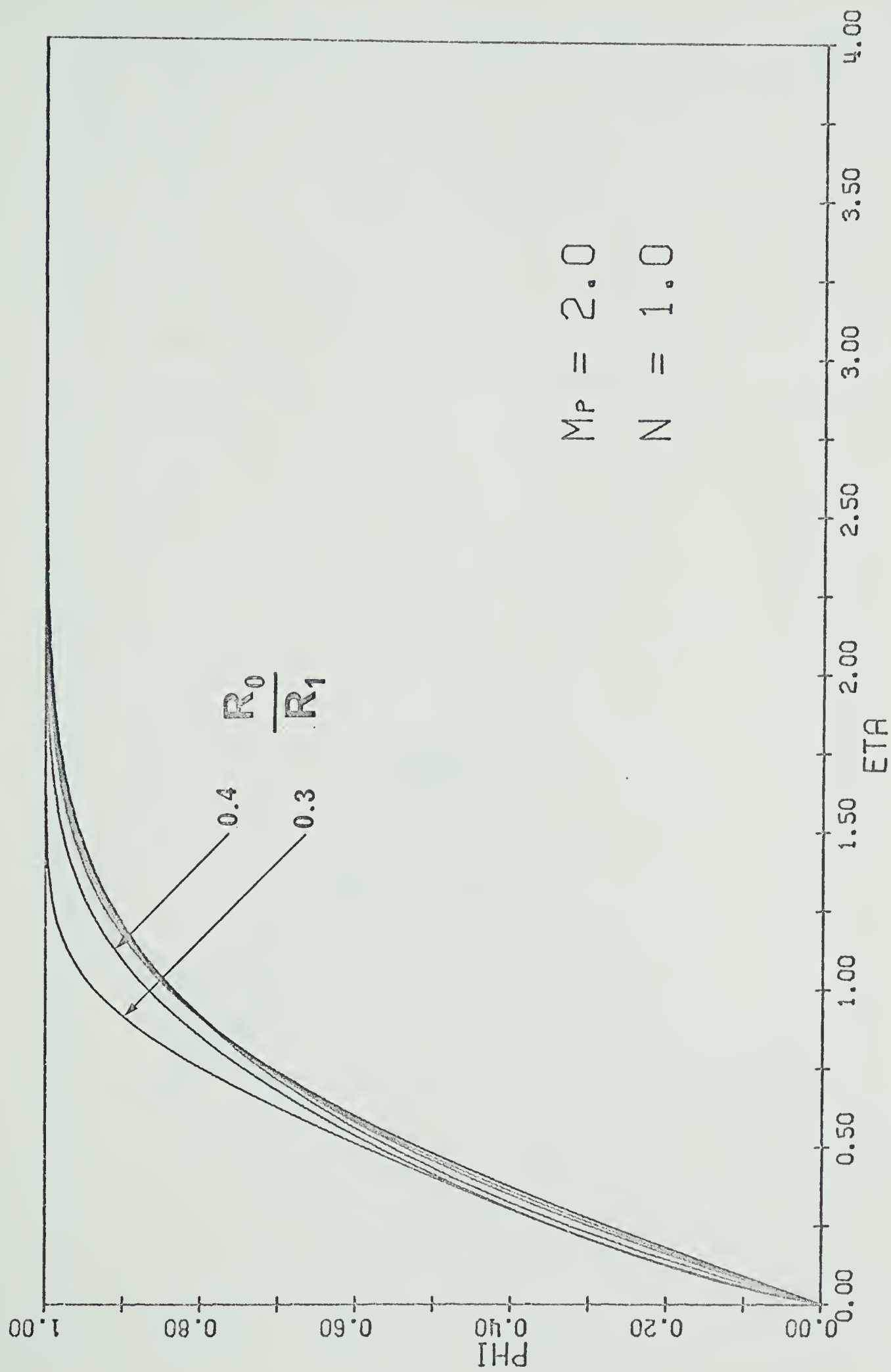


FIG. 4.1.e ϕ VS η , NEWTONIAN FLUID, $M_p = 2.0$

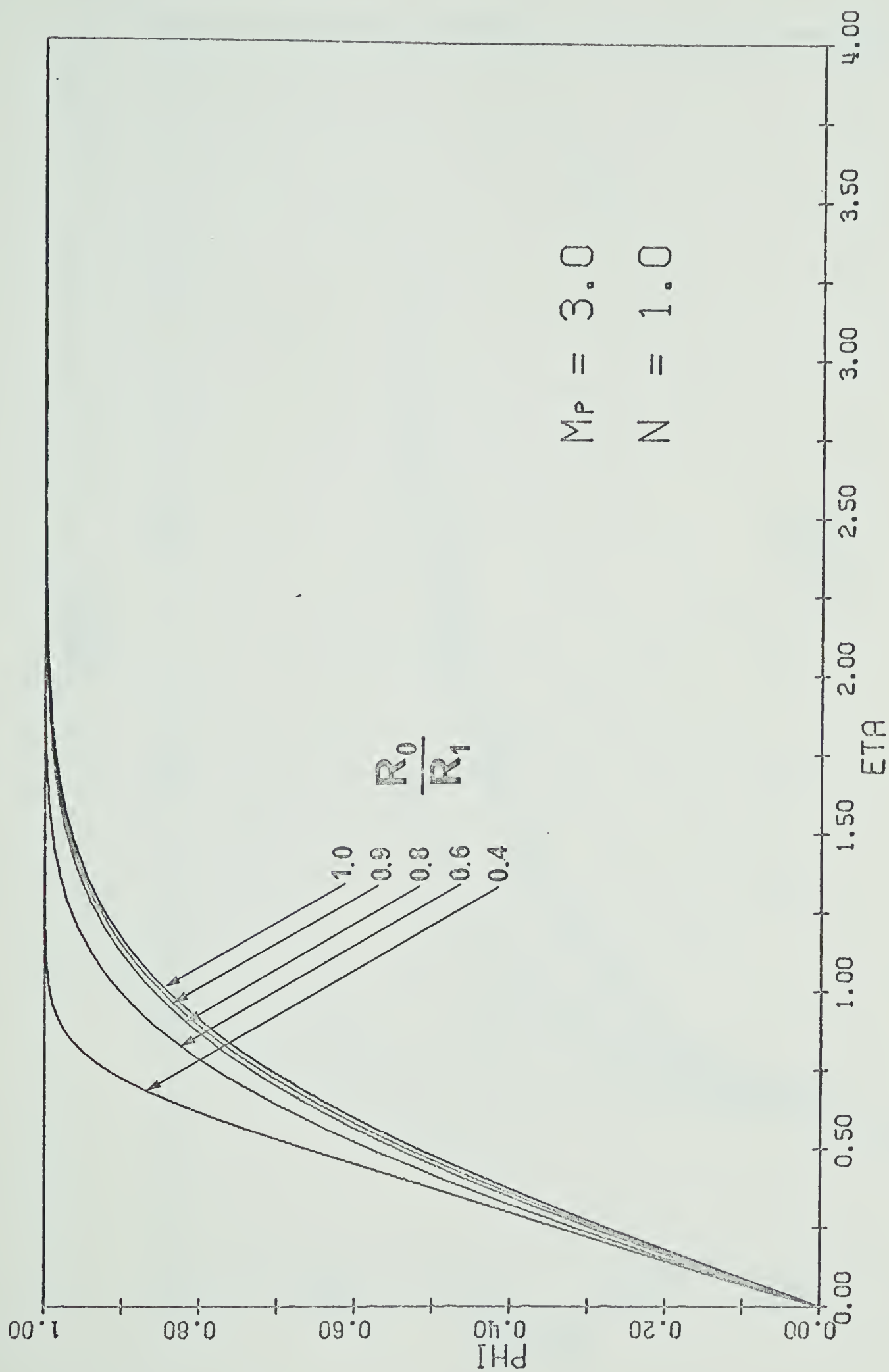


FIG. 4.1.1.f ϕ vs η , NEWTONIAN FLUID, $M_P = 3.0$

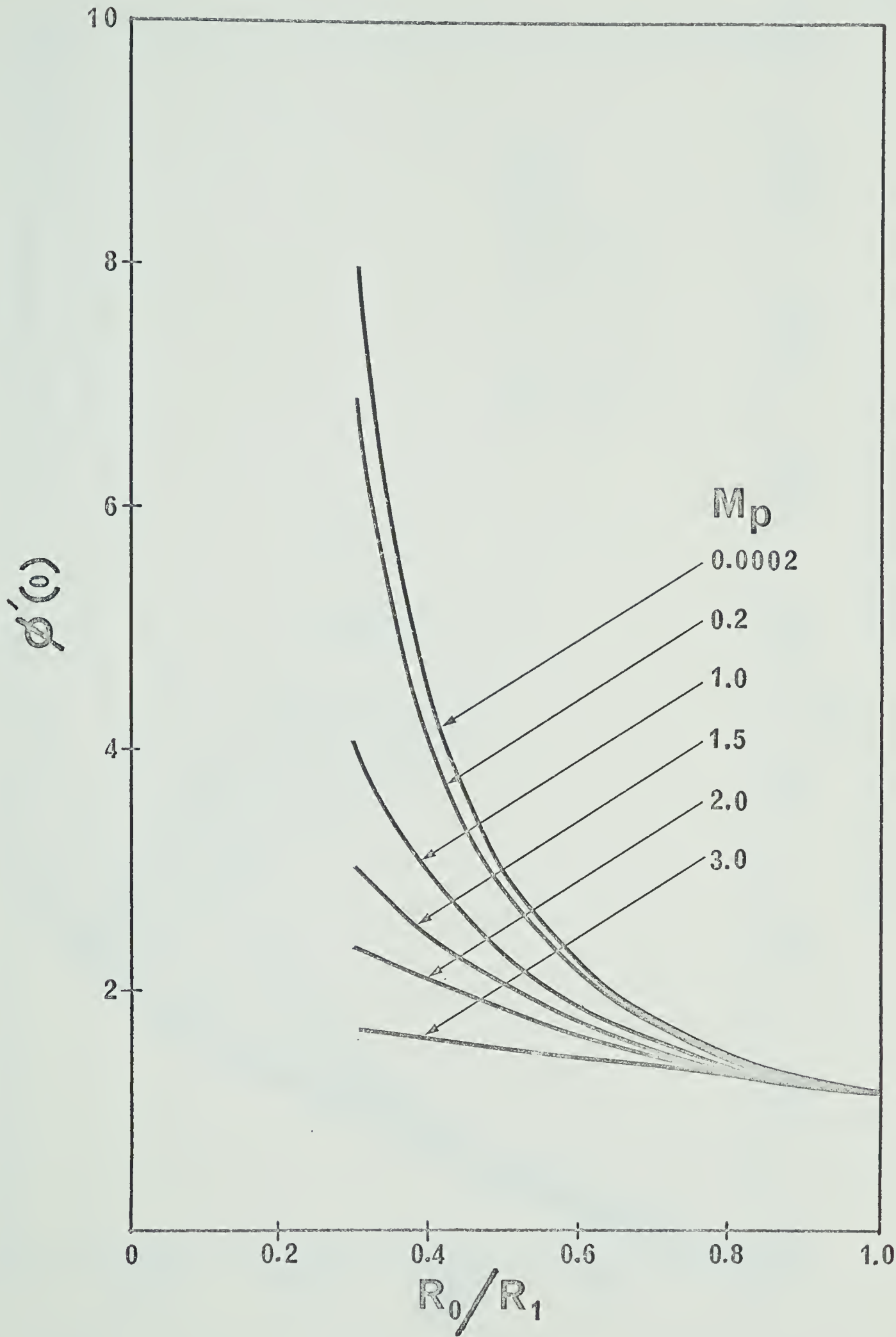


FIG. 4.2 EFFECT OF WALL RHEOLOGY AND DILATATION RATIO ON EXPULSION RATE FOR A YIELDING MATERIAL, $N = 1$

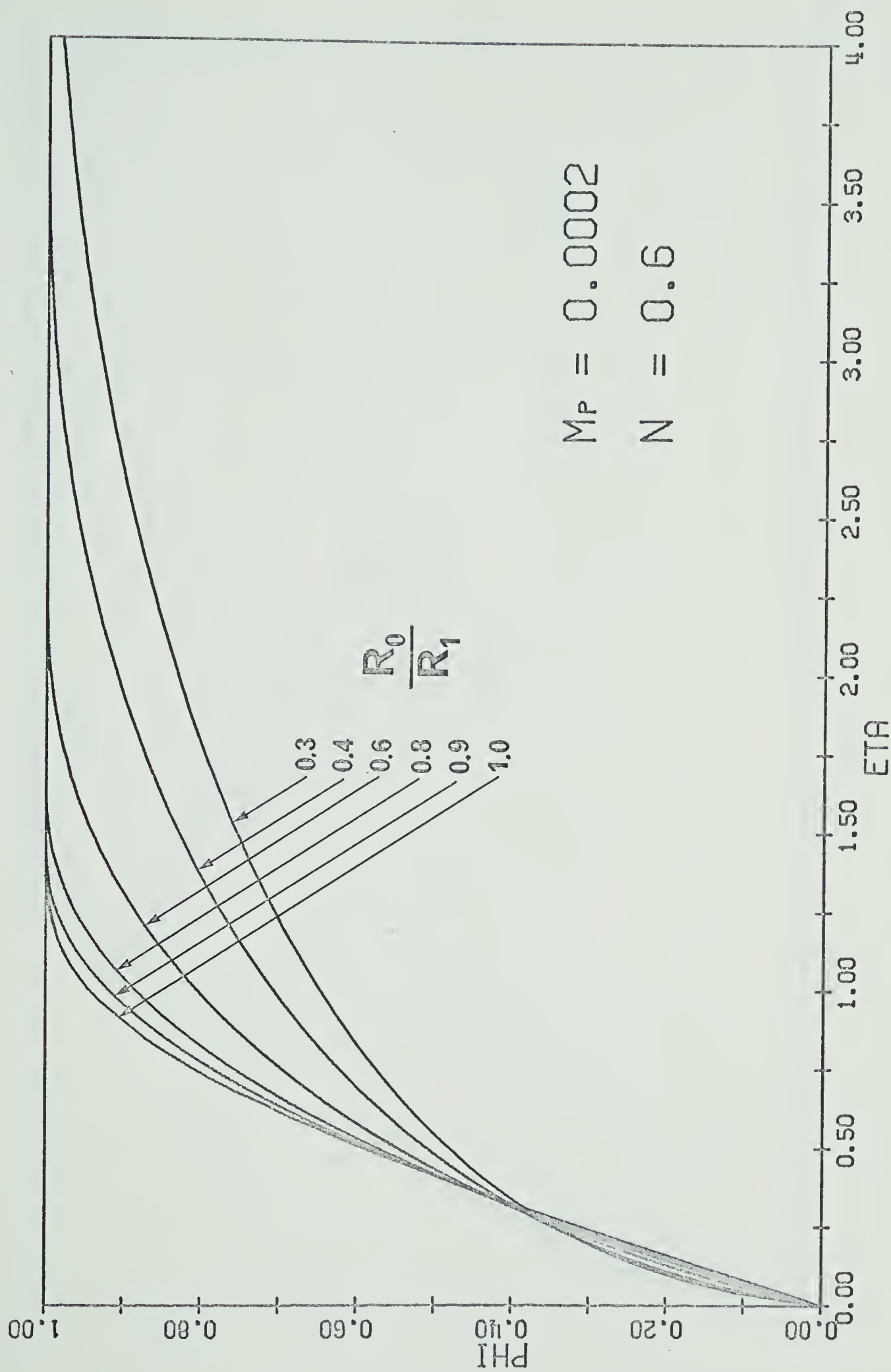


FIG. 4.3.a ϕ VS η , LINEAR WALL, $N = 0.6$

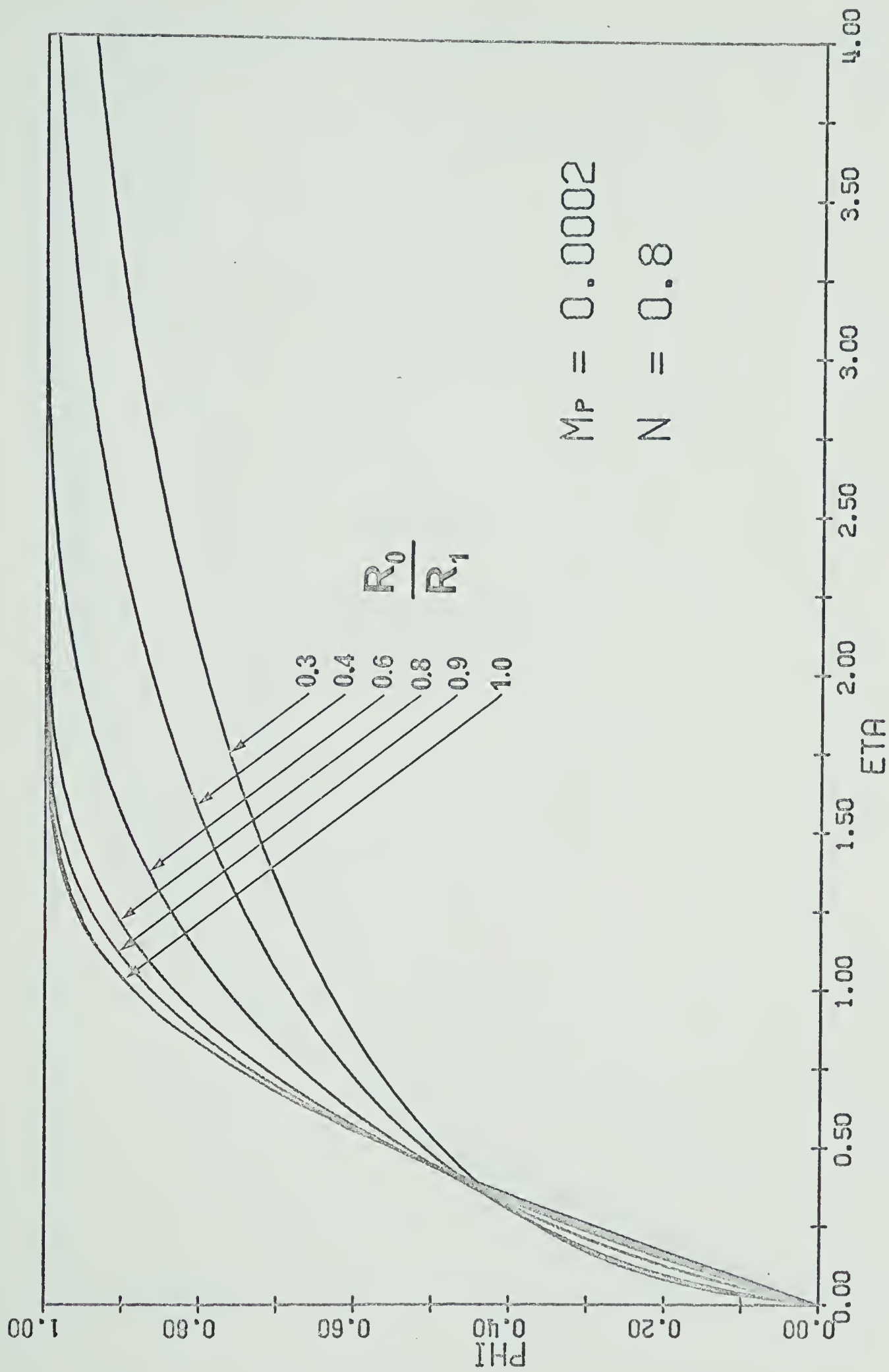


FIG. 4.3.b ϕ VS η , LINEAR WALL, $N = 0.8$

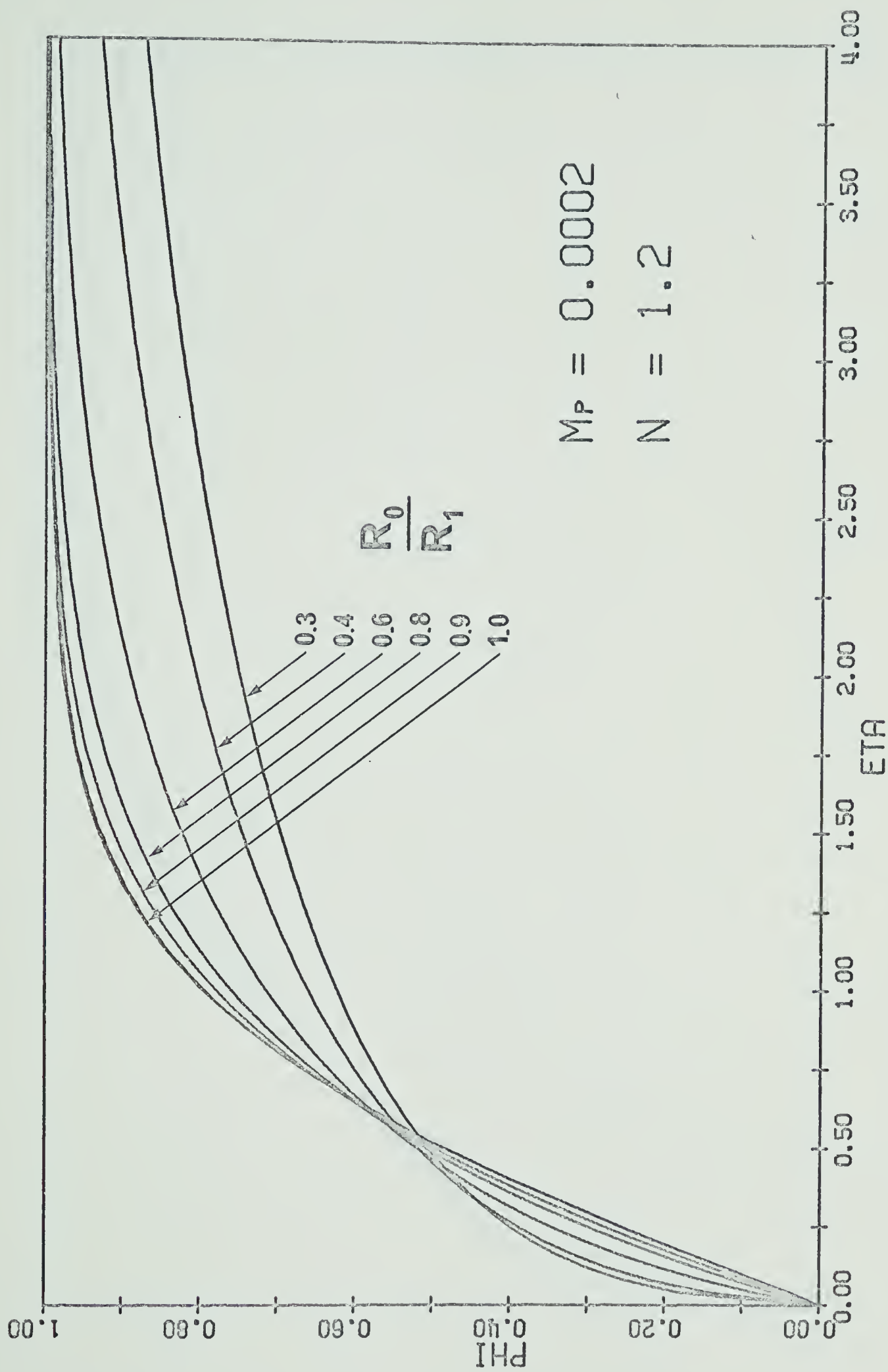


FIG. 4.3.c ϕ VS η , LINEAR WALL, $N = 1.2$

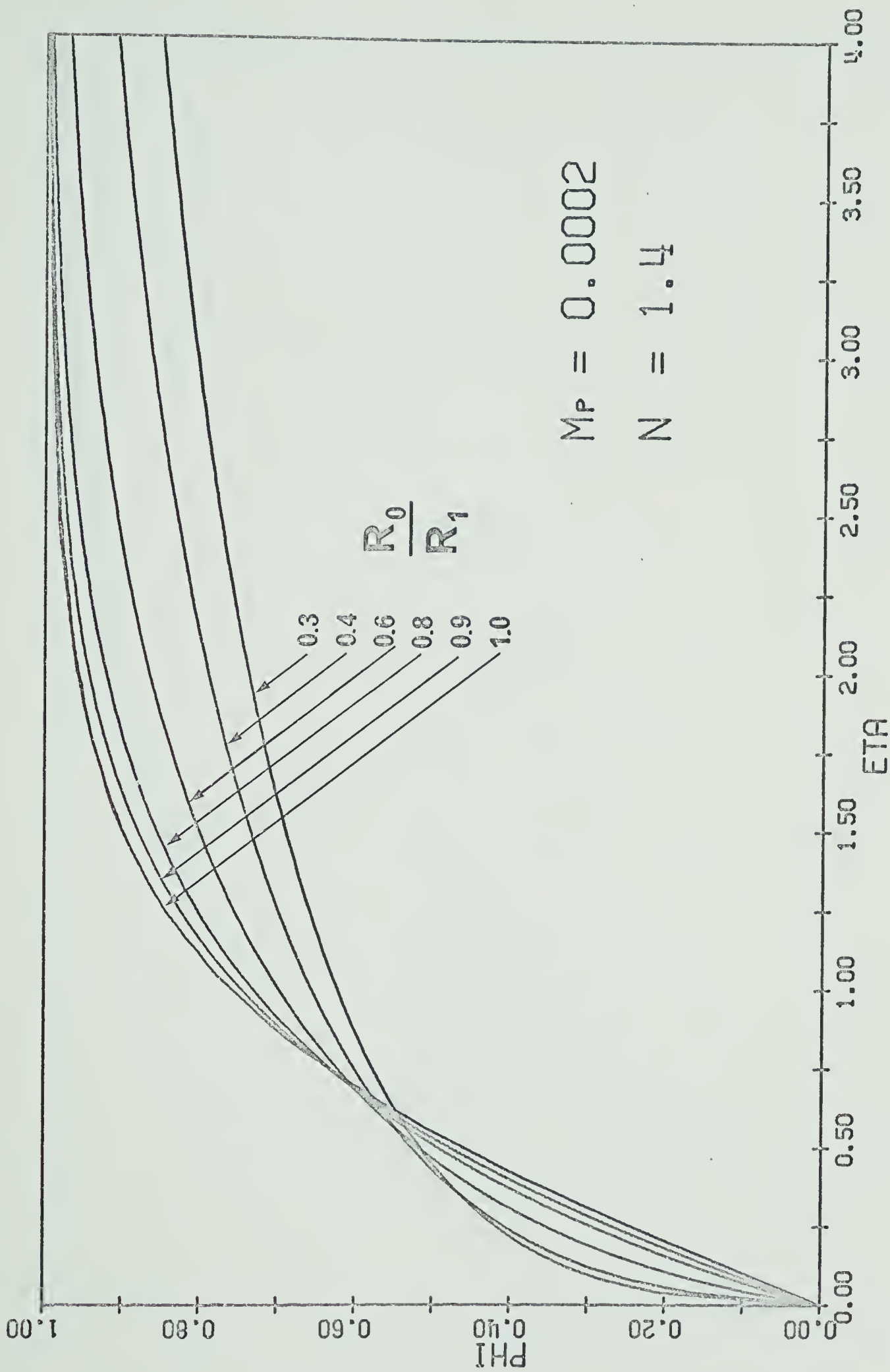


FIG. 4.3.d ϕ VS η , LINEAR WALL, $N = 1.4$

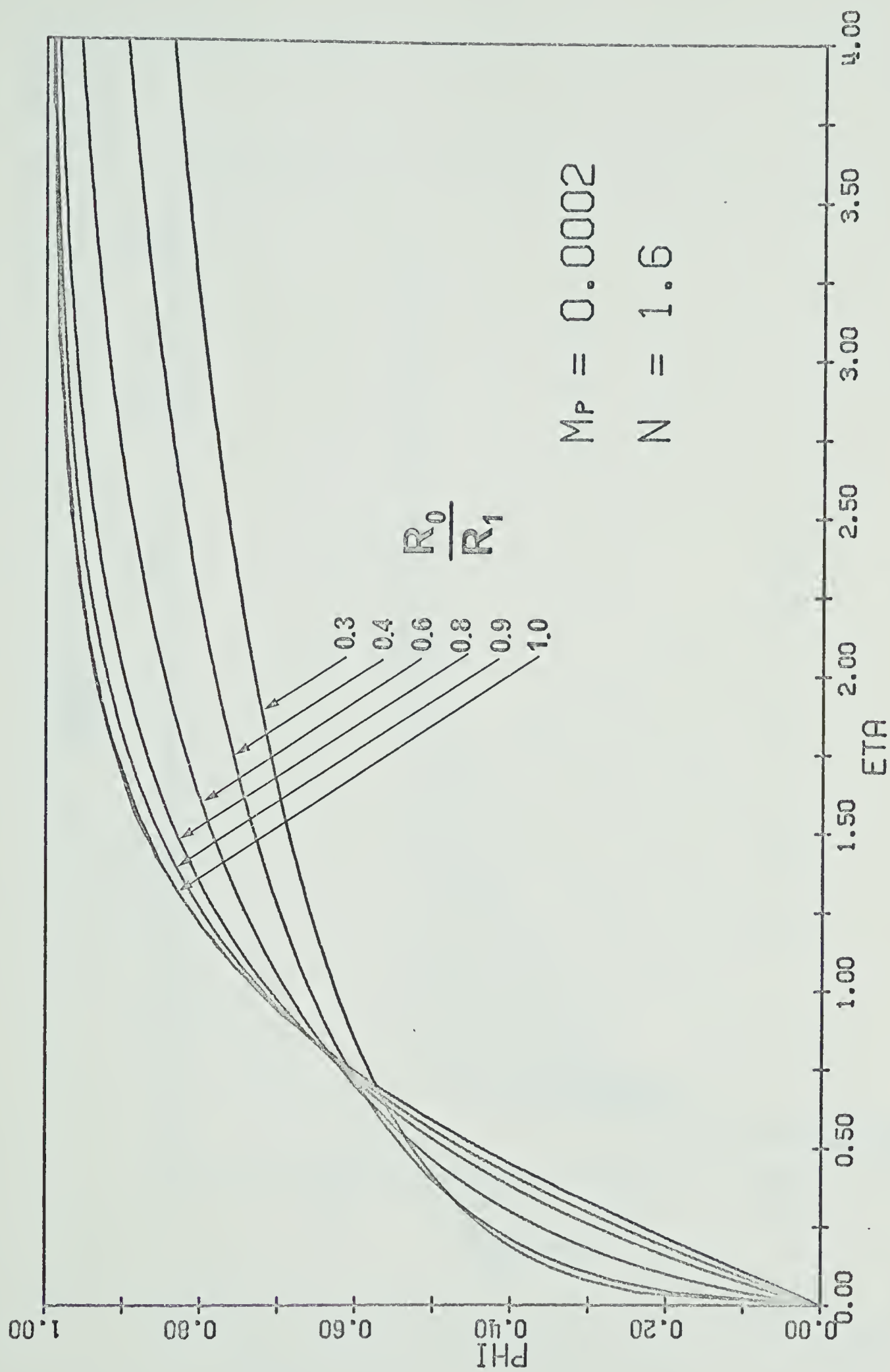


FIG. 4.3.e ϕ VS η , LINEAR WALL, $N = 1.6$

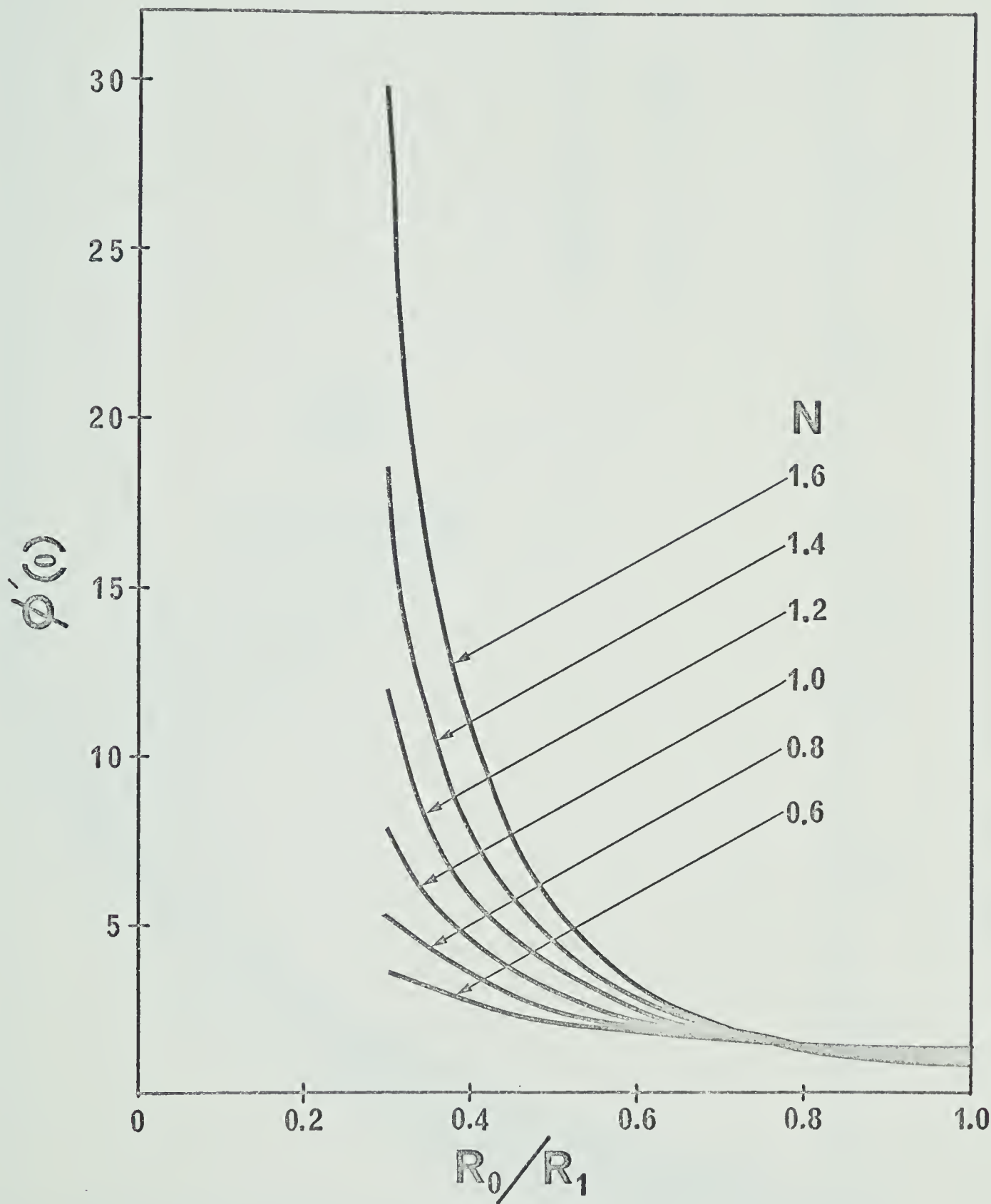


FIG. 4.4 EFFECT OF FLUID RHEOLOGY AND DILATATION RATIO ON EXPLUSION RATE FOR A YIELDING MATERIAL $M_p = 0.0002$

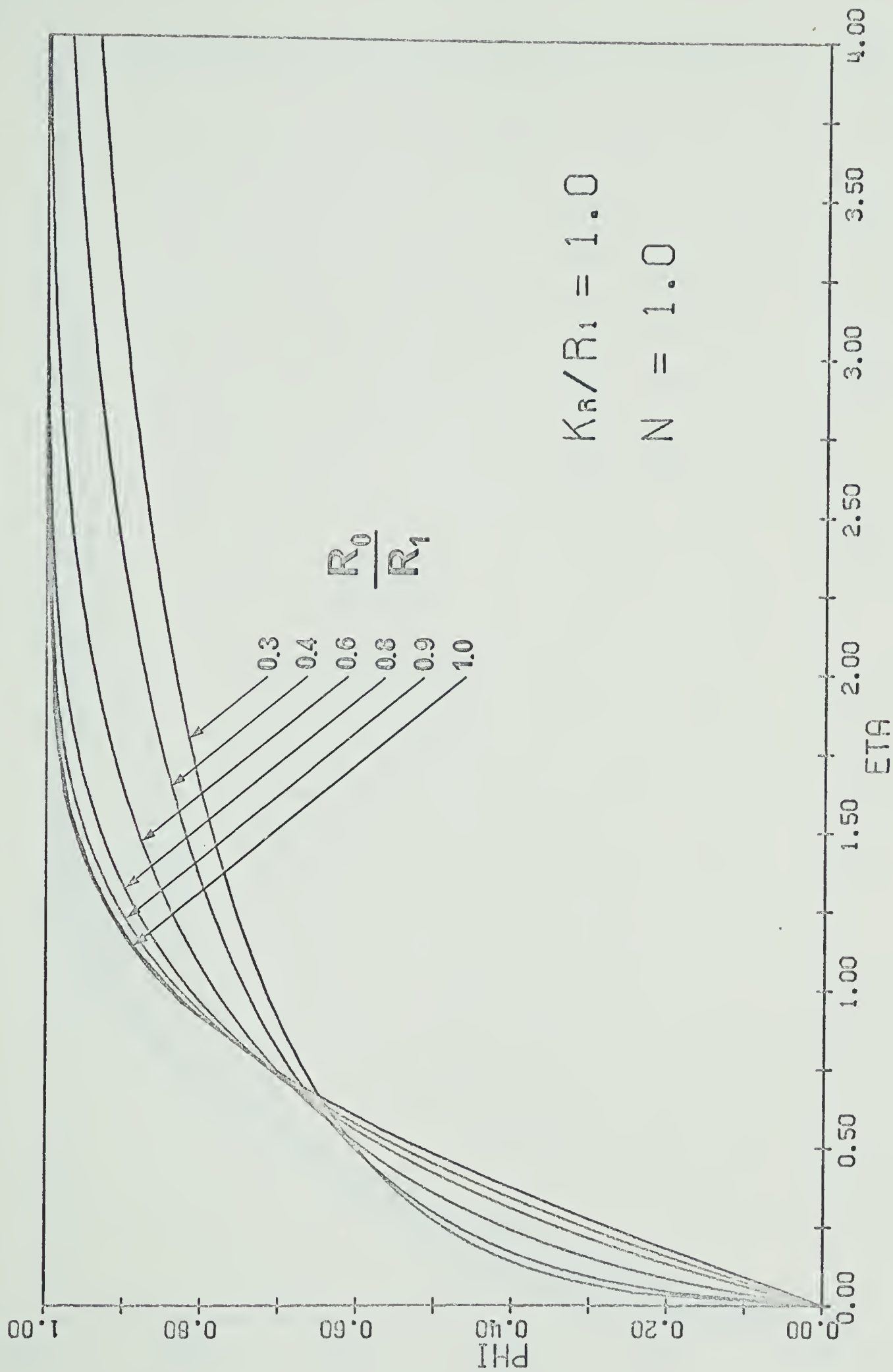


FIG. 4.5.a ϕ VS η , NEWTONIAN FLUID, $K_R/R_1 = 1.0$

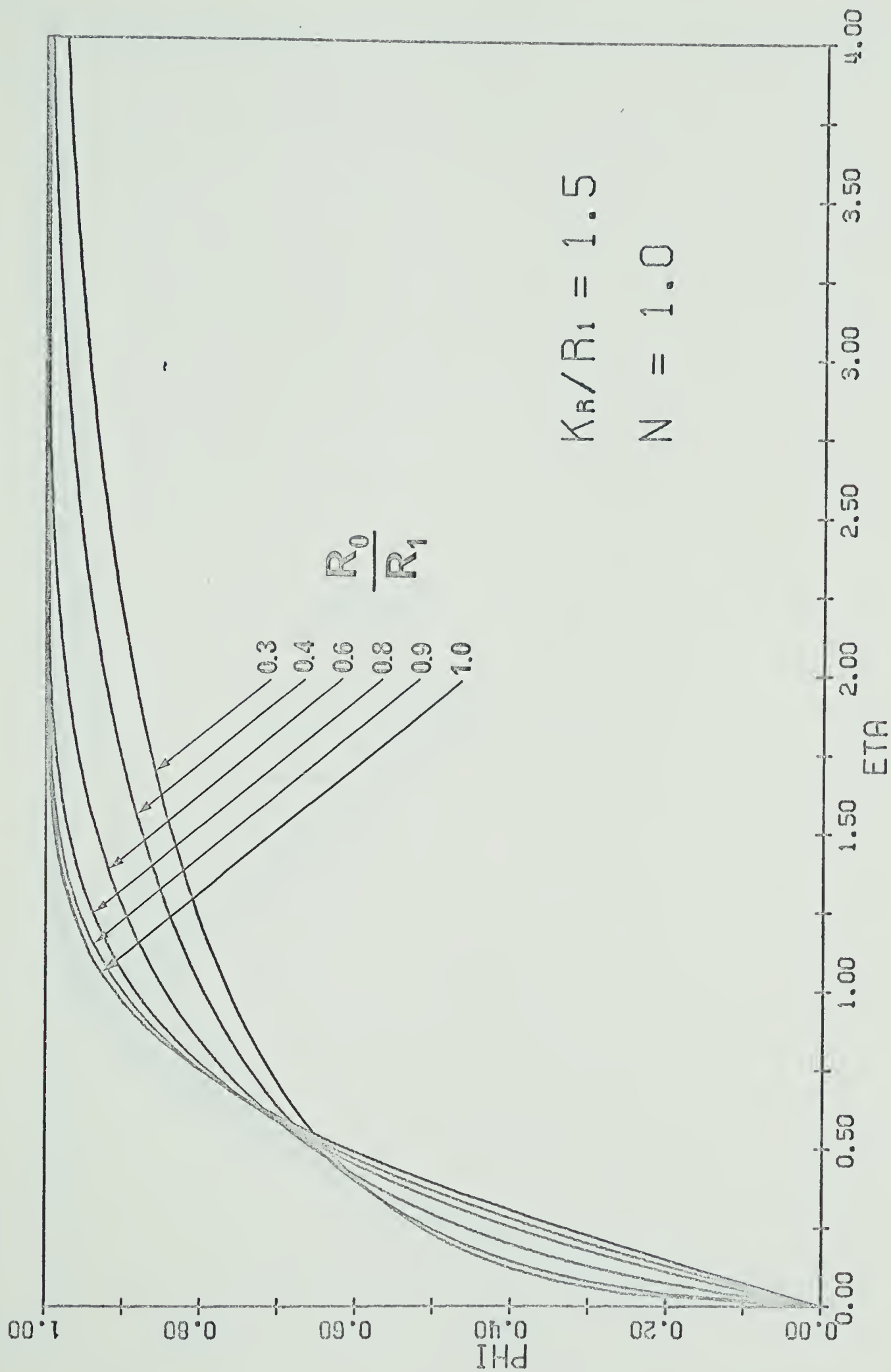


FIG. 4.5.b ϕ VS η , NEWTONIAN FLUID, $K_R/R_1 = 1.5$

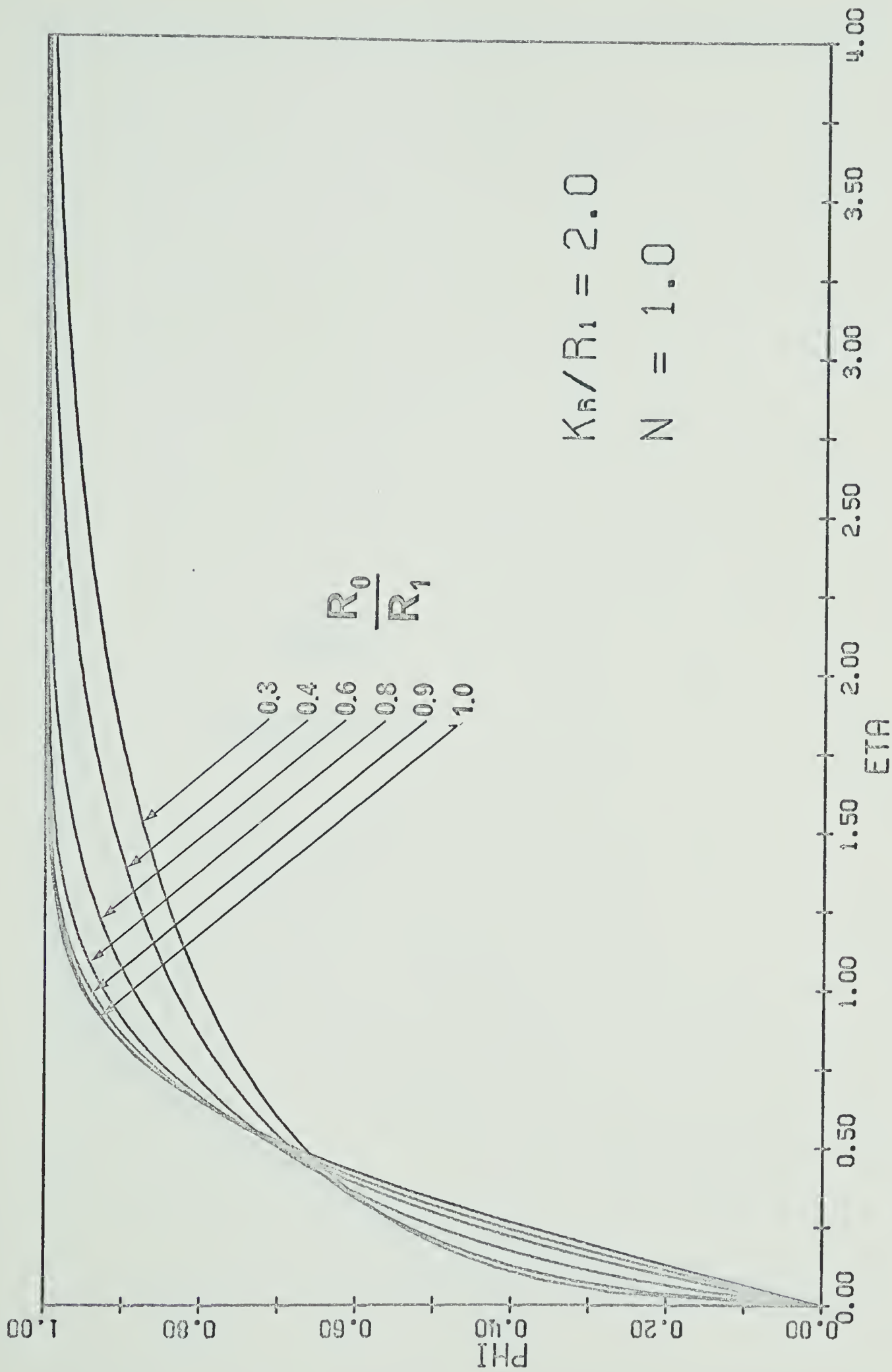


FIG. 4.5.c ϕ VS η , NEWTONIAN FLUID, $K_R/R_1 = 2.0$

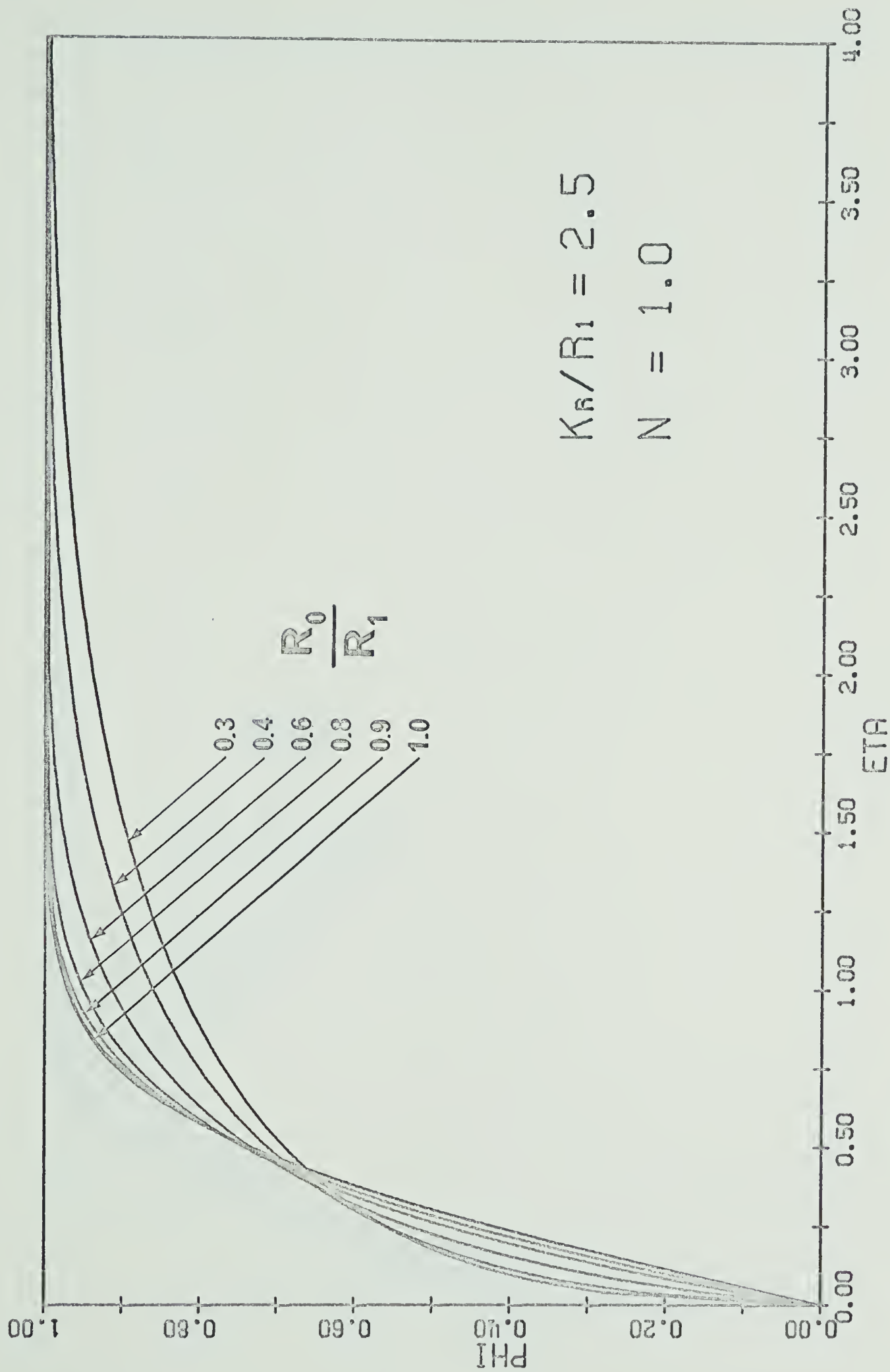


FIG. 4.5.d ϕ VS η , NEWTONIAN FLUID, $K_R/R_1 = 2.5$

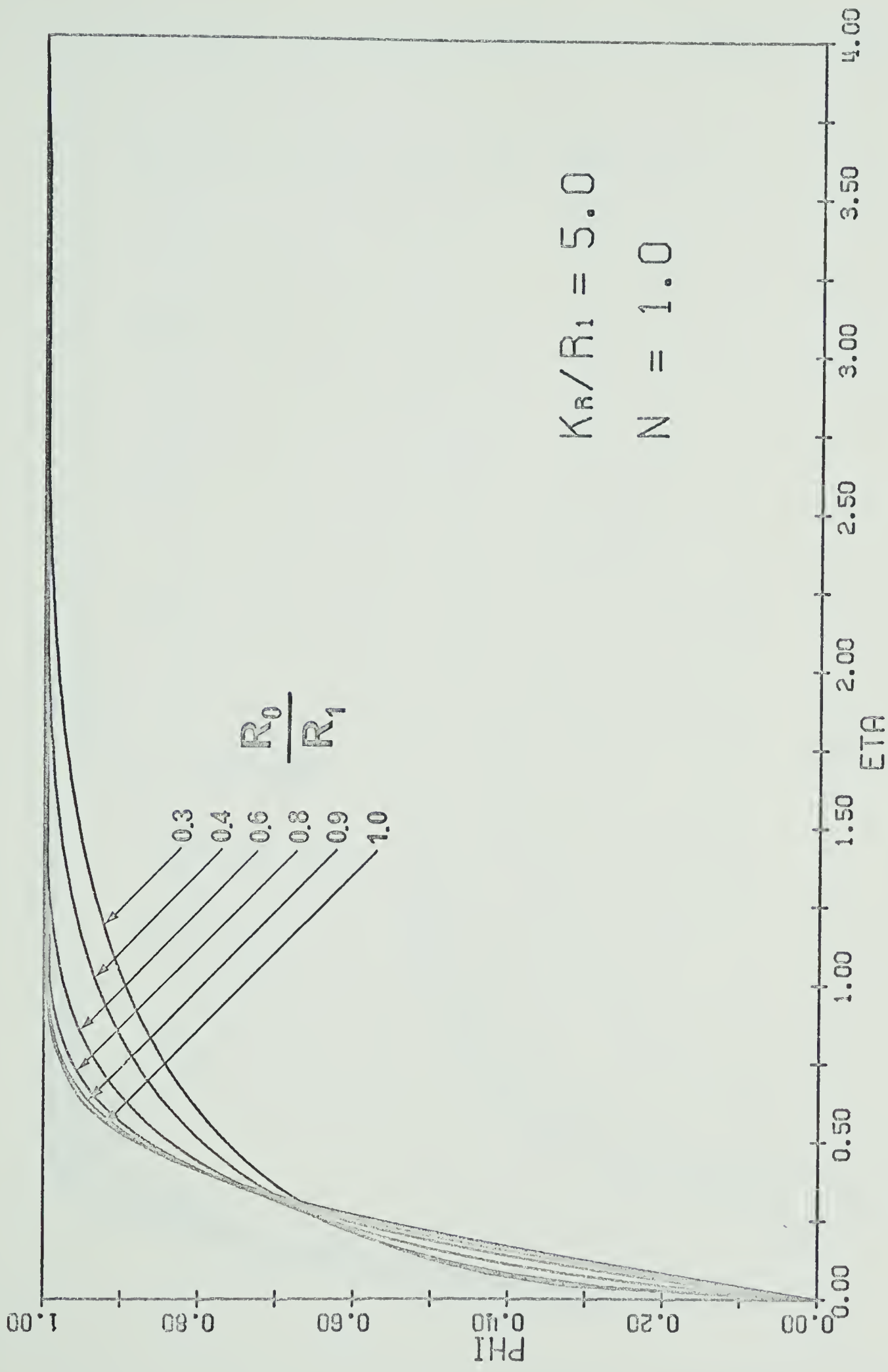


FIG. 4.5.e ϕ VS η , NEWTONIAN FLUID, $K_R/R_1 = 5.0$

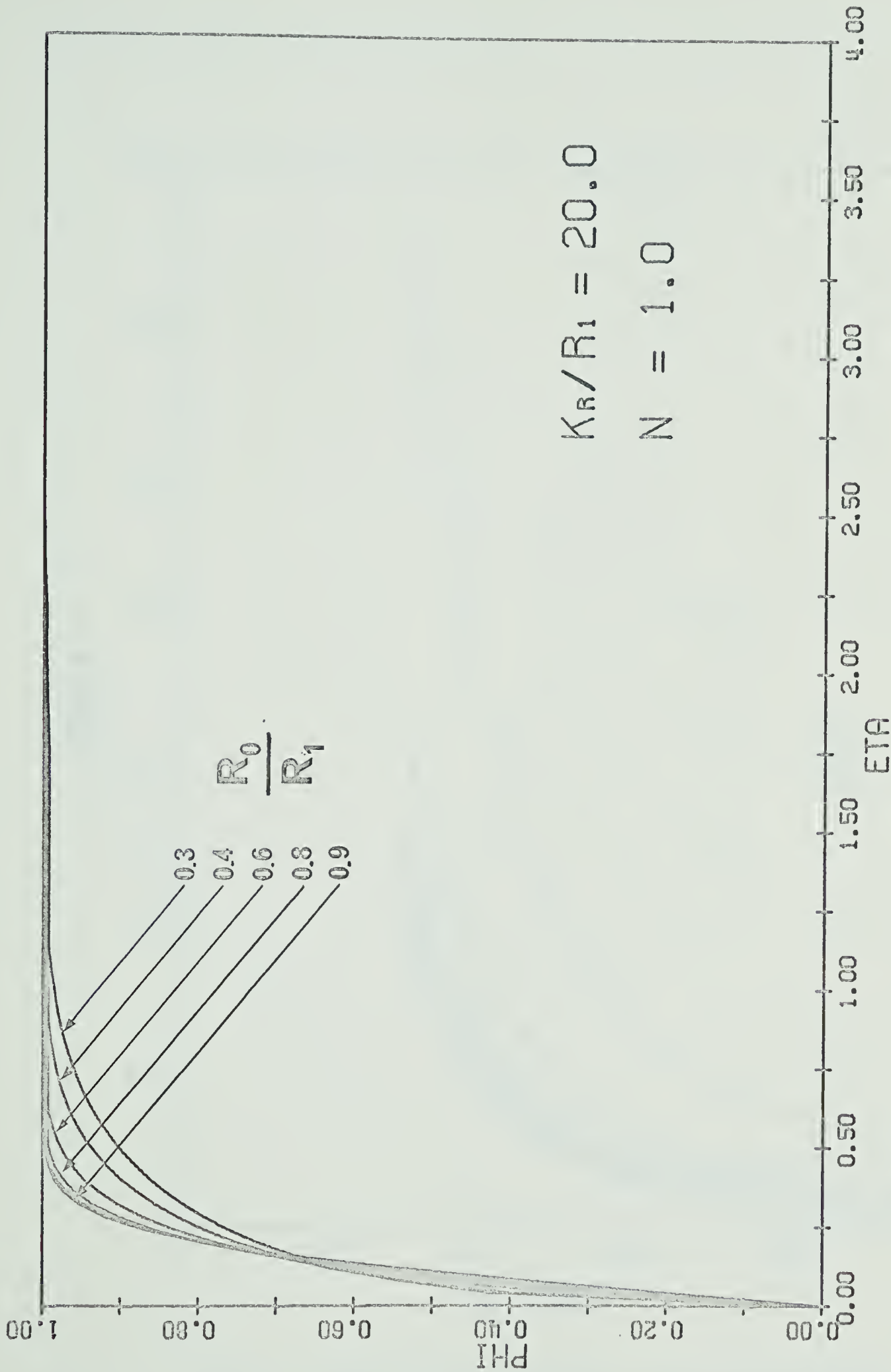


FIG. 4.5.f ϕ VS η , NEWTONIAN FLUID, $K_R/R_1 = 20.0$

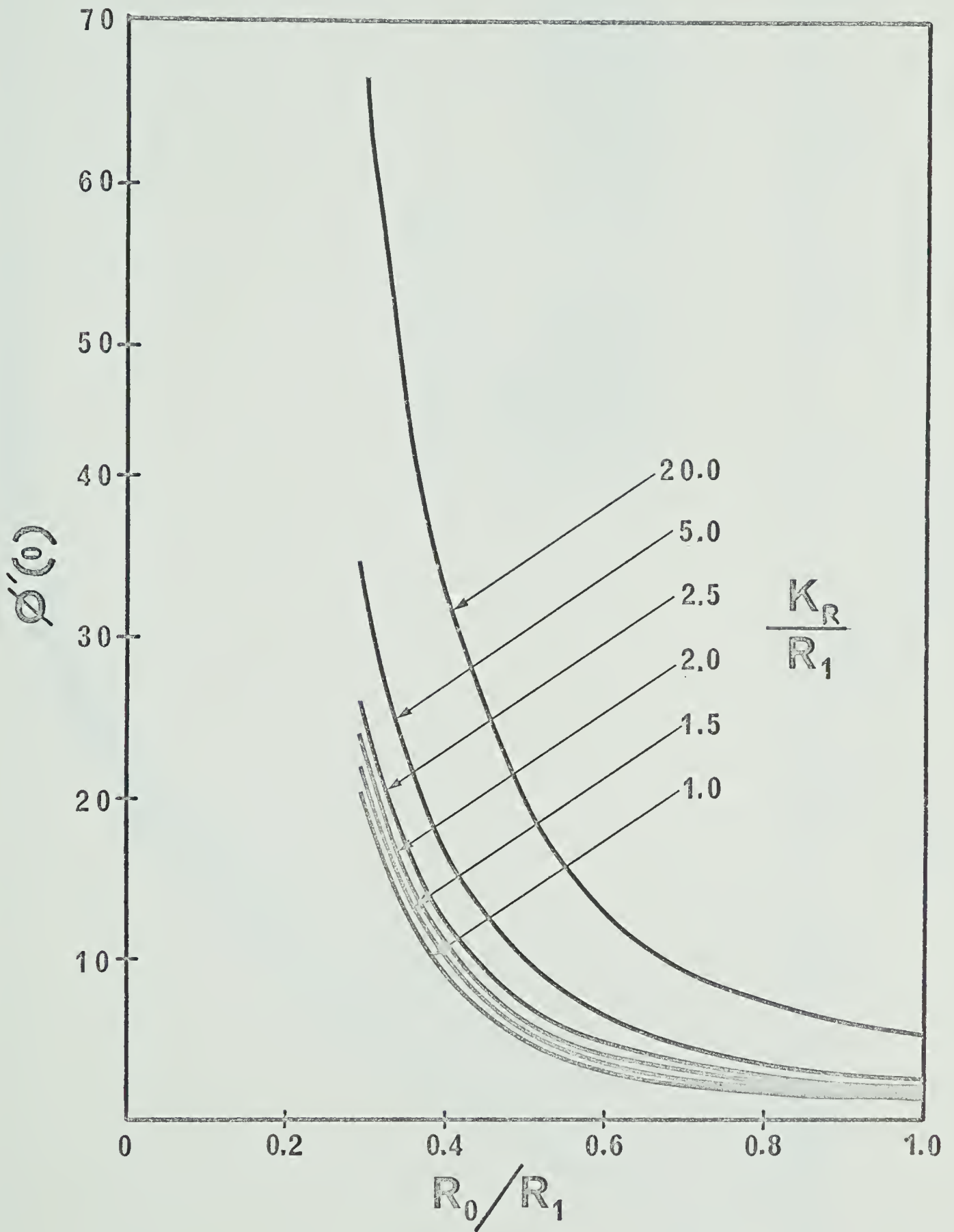


FIG. 4.6 EFFECT OF WALL RHEOLOGY AND DILATATION RATIO ON EXPULSION RATE FOR A LOCKING MATERIAL

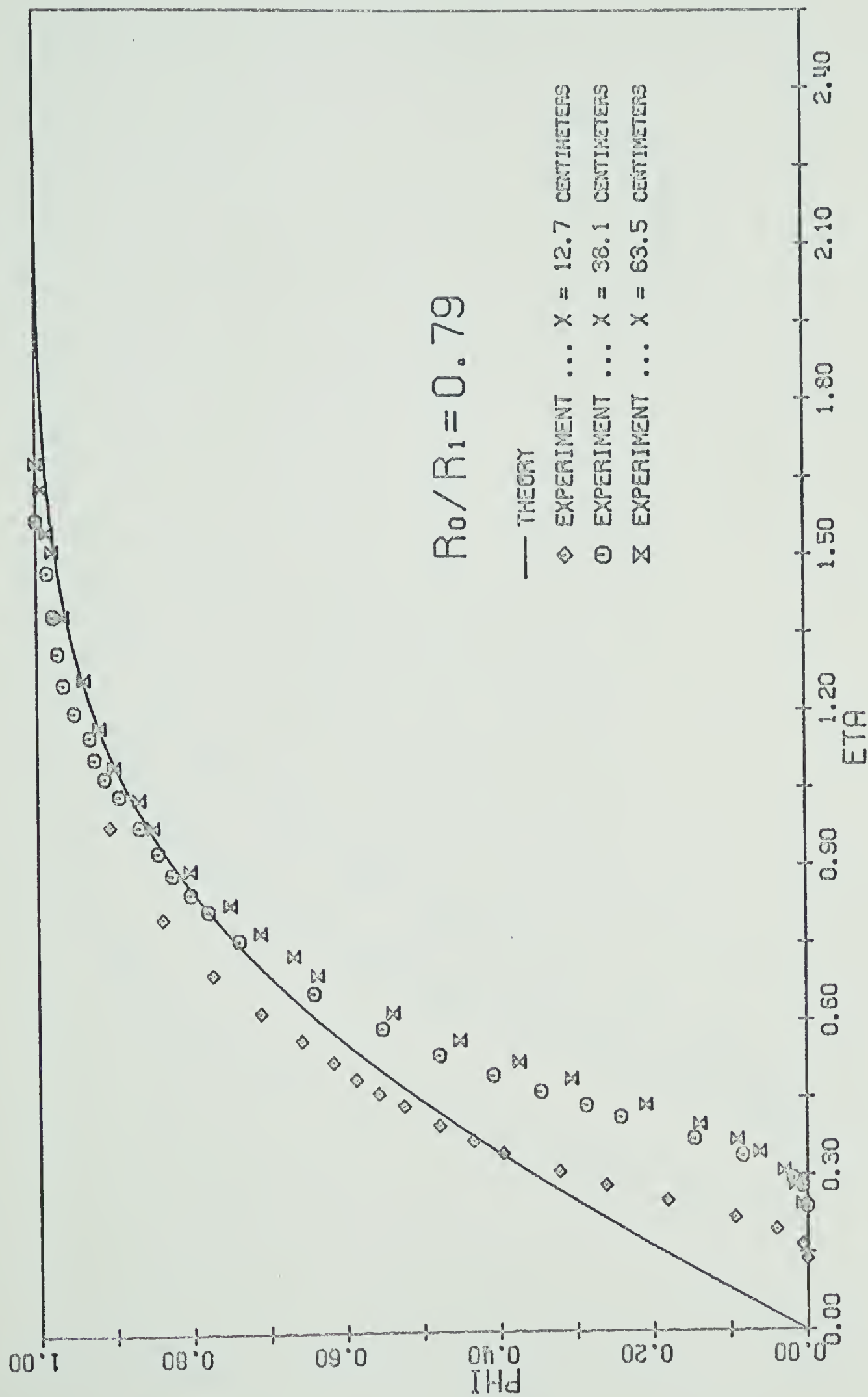


FIG. 4.7.a ϕ VS η , $R_0/R_1 = 0.79$, $\mu = 4.26$ POISE

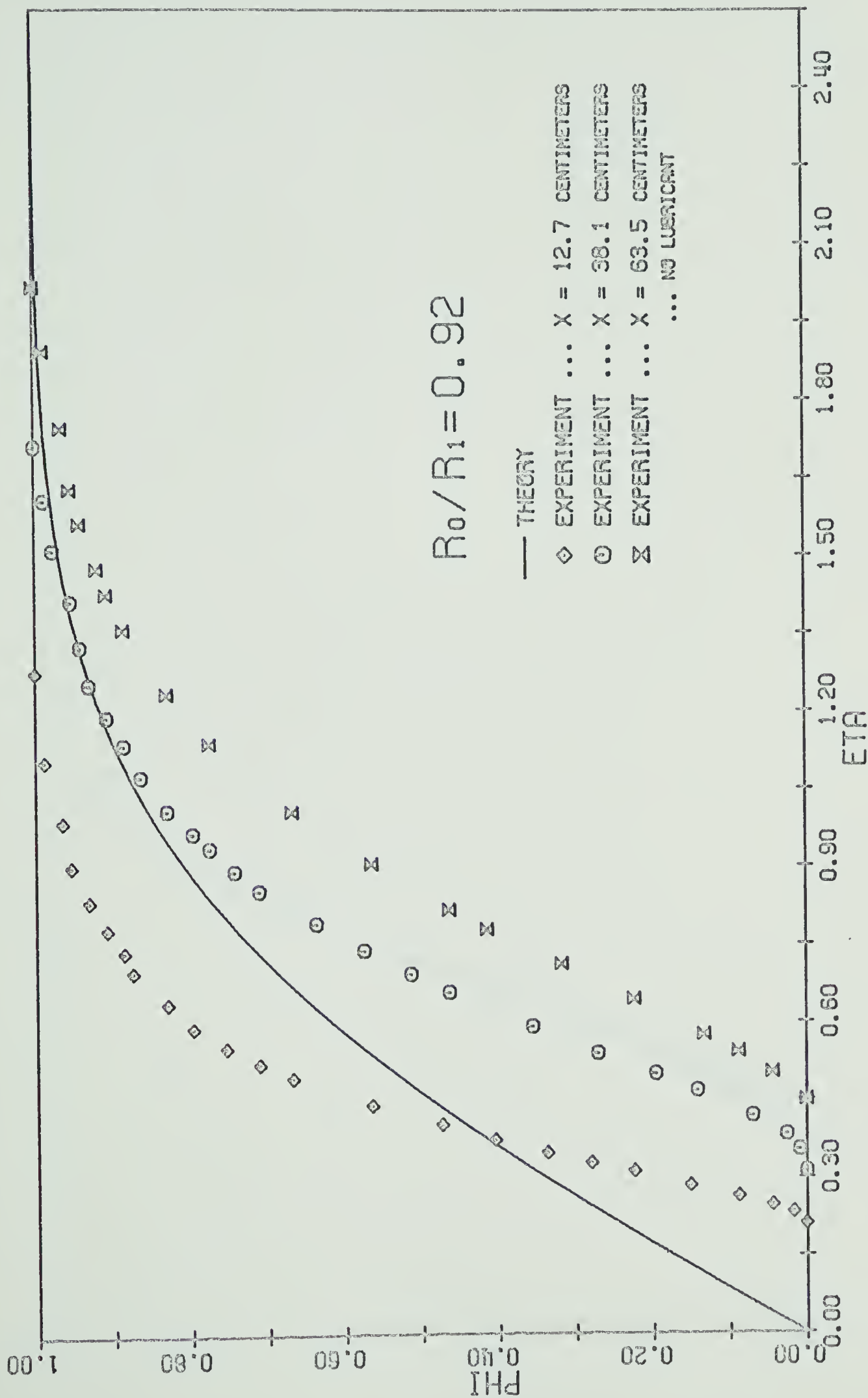


FIG. 4.7.b ϕ VS η , $R_0/R_1 = 0.92$, $\mu = 4.26$ POISE
EFFECT OF NO LUBRICANT

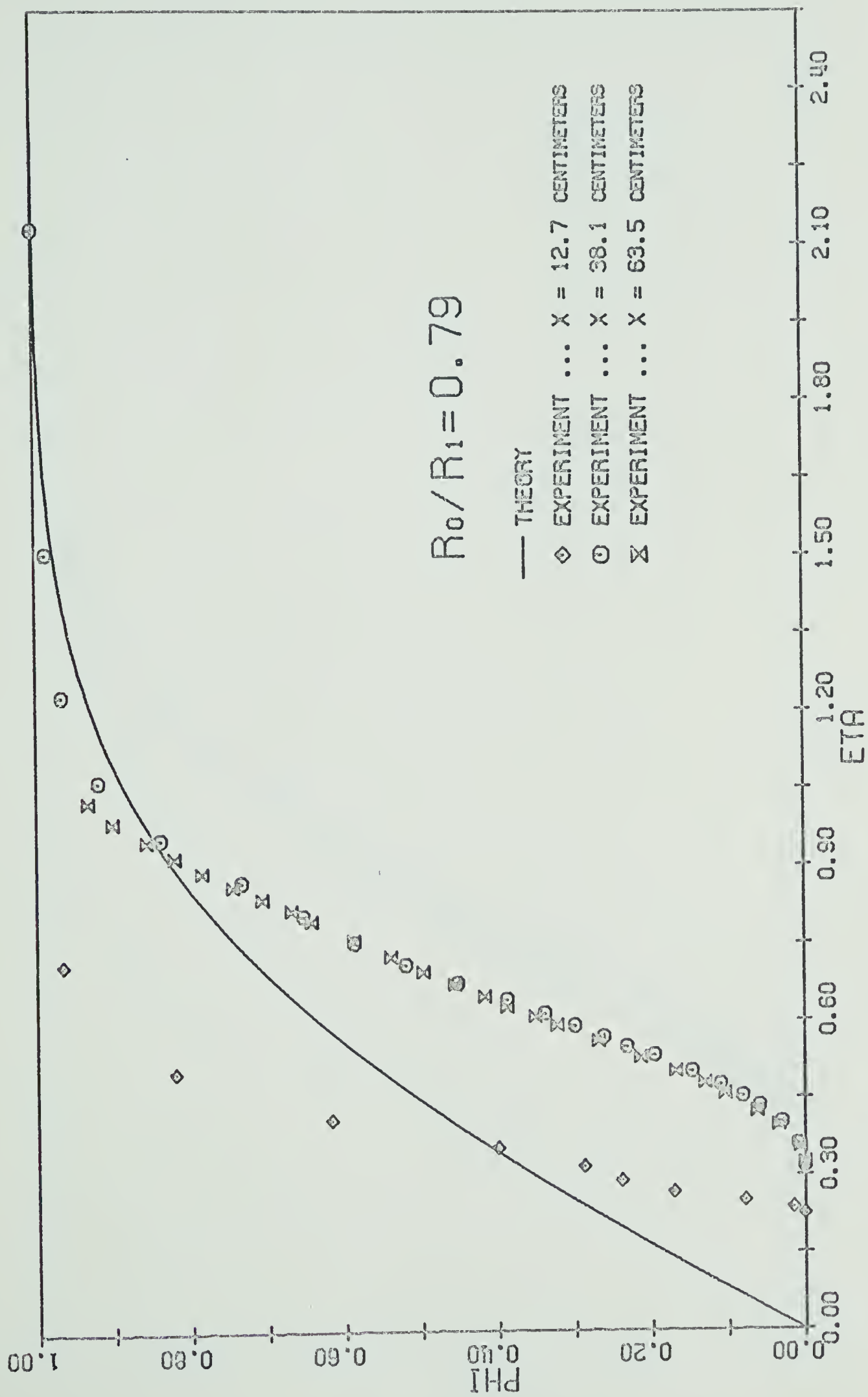


FIG. 4.7.c ϕ VS η , $R_0/R_1 = 0.79$, $\mu = 1.12$ POISE

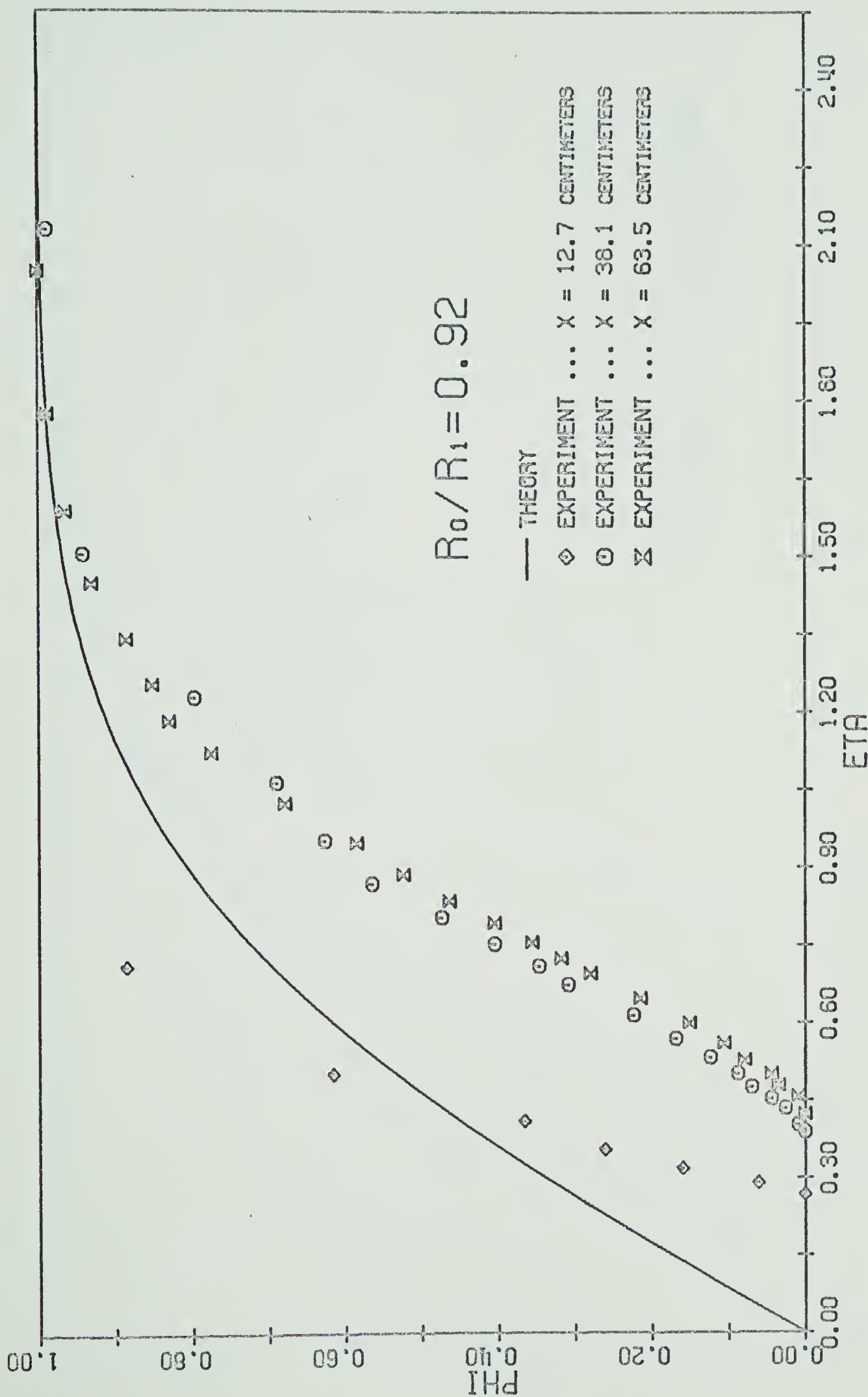


FIG. 4.7.d ϕ VS η , $R_0/R_1 = 0.92$, $\mu = 1.12$ POISE

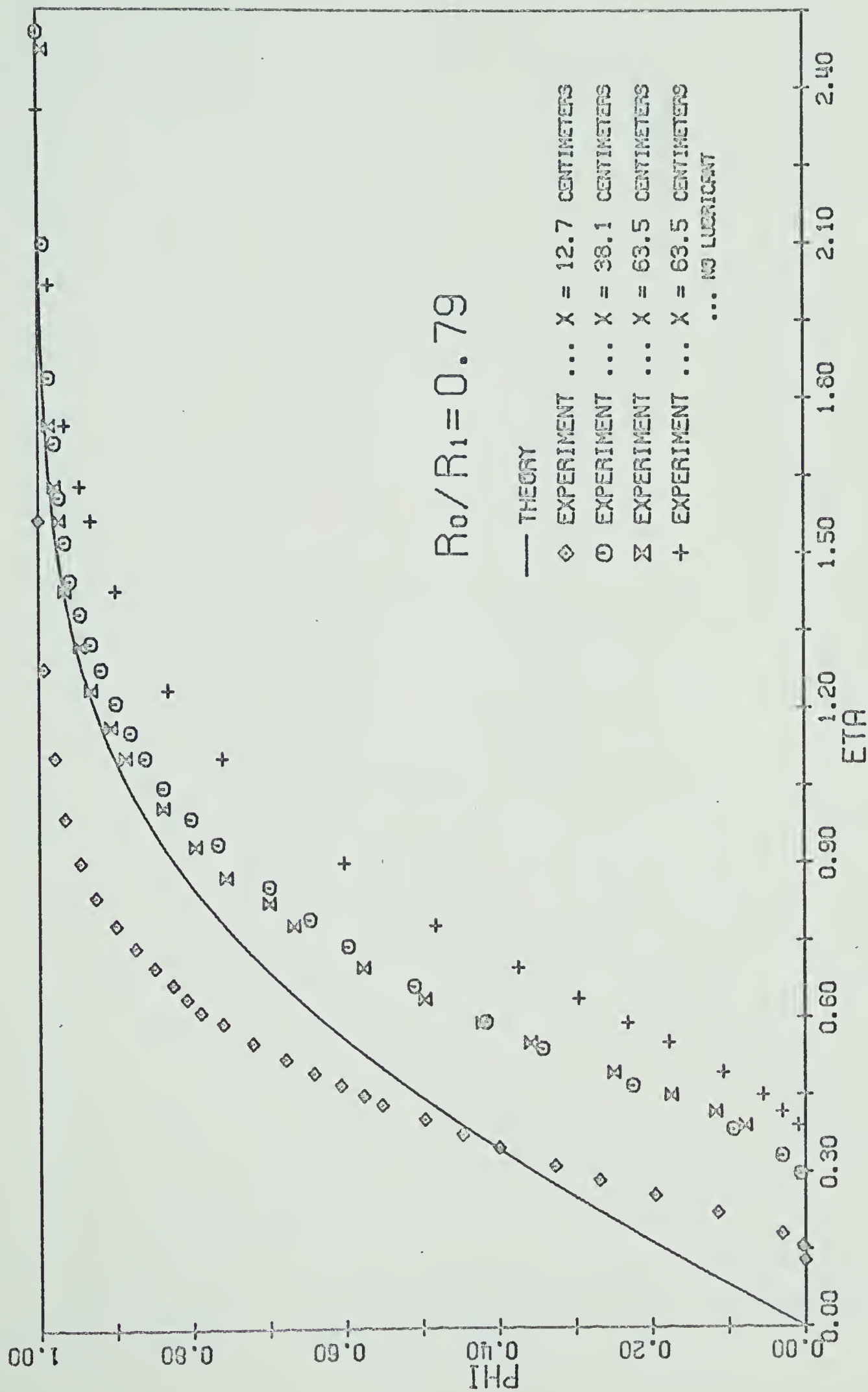


FIG. 4.7.e ϕ VS η , $R_0/R_1 = 0.79$, $\mu = 10.80$ POISE

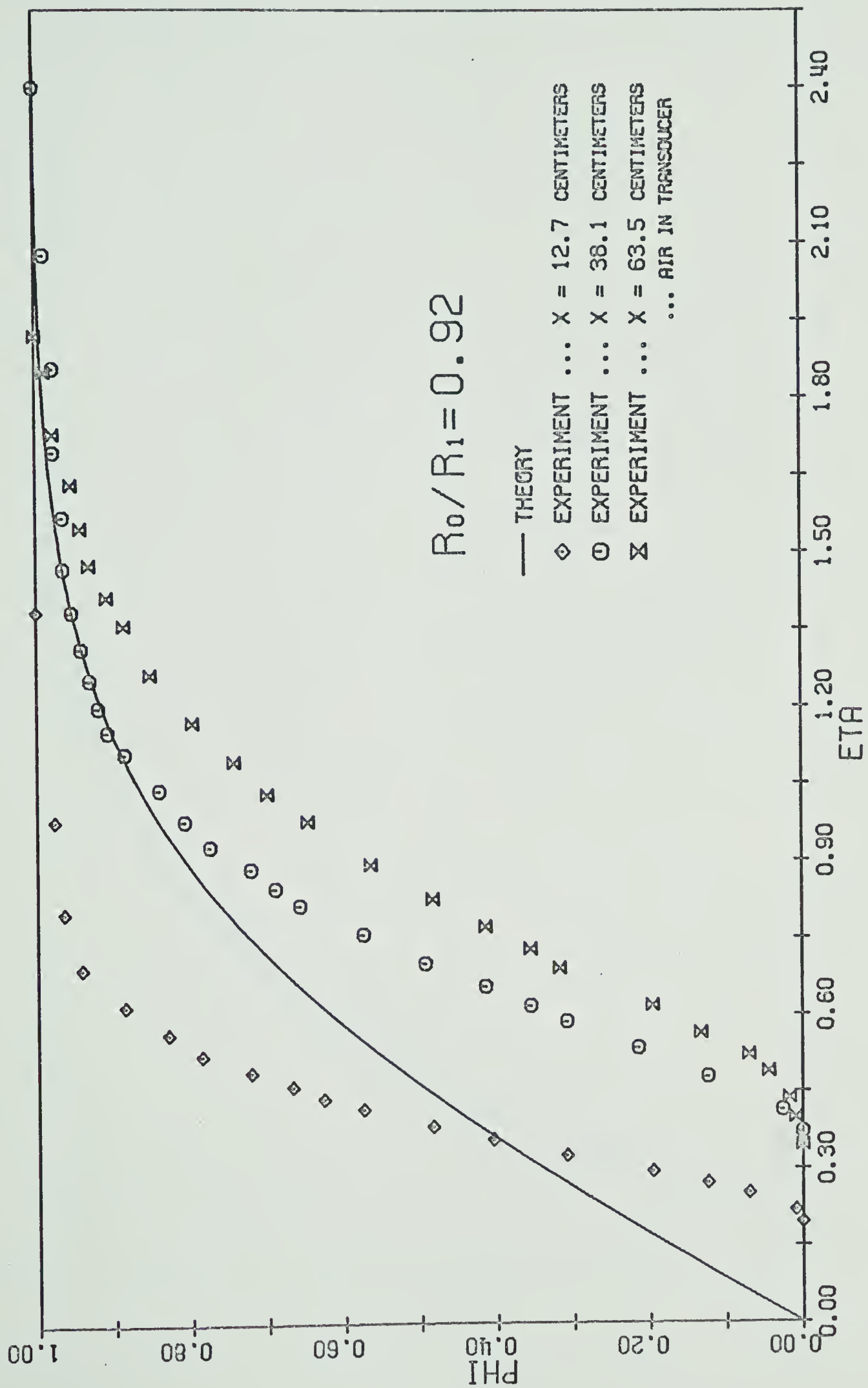


FIG. 4.7.f ϕ VS η , $R_0/R_1 = 0.92$, $\mu = 10.80$ POISE
EFFECT OF AIR BUBBLES

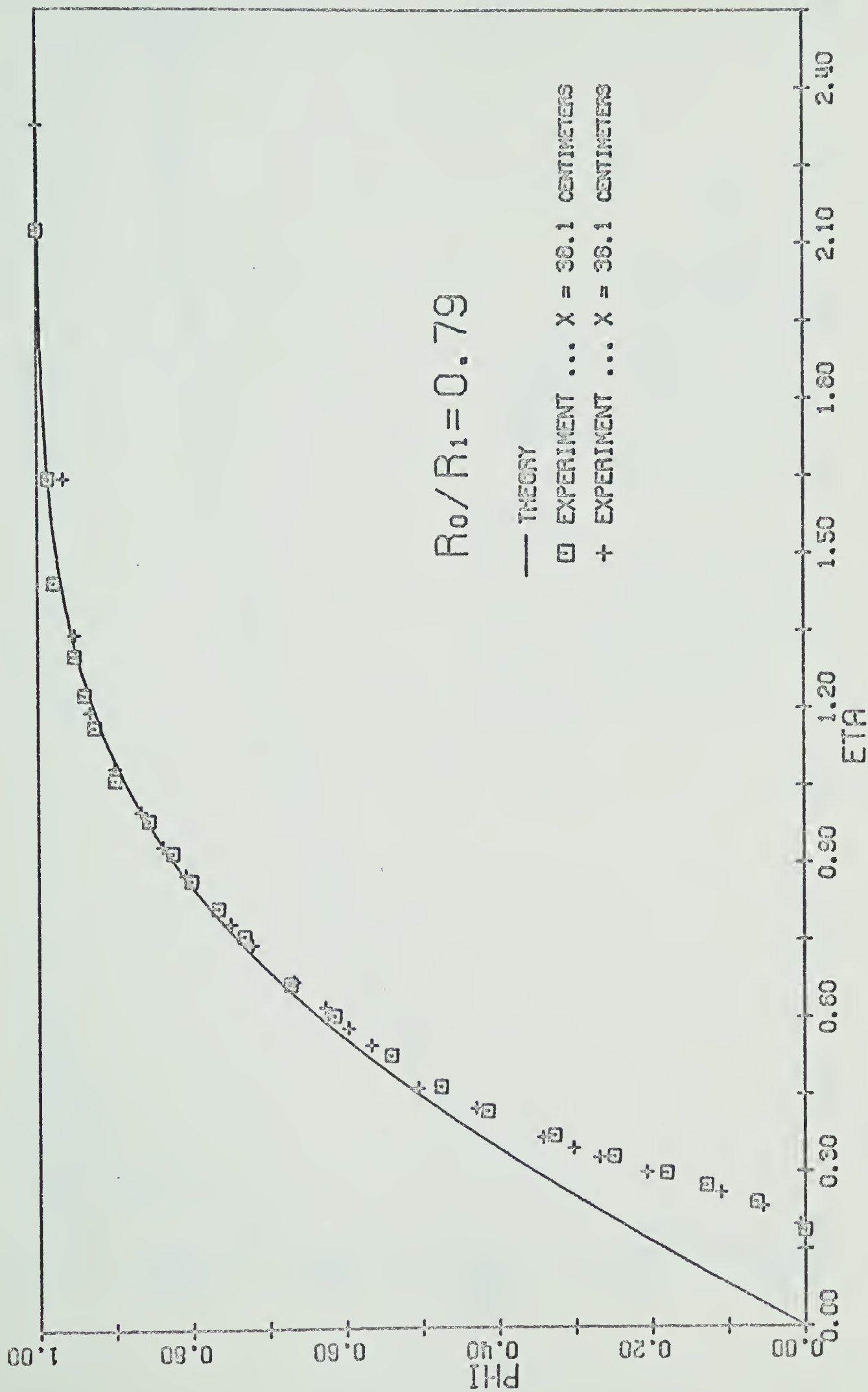


FIG. 4.7.g ϕ VS η , $R_0/R_1 = 0.79$, $\mu = 10.80$ POISE
 "SEMI-INFINITE TUBE"

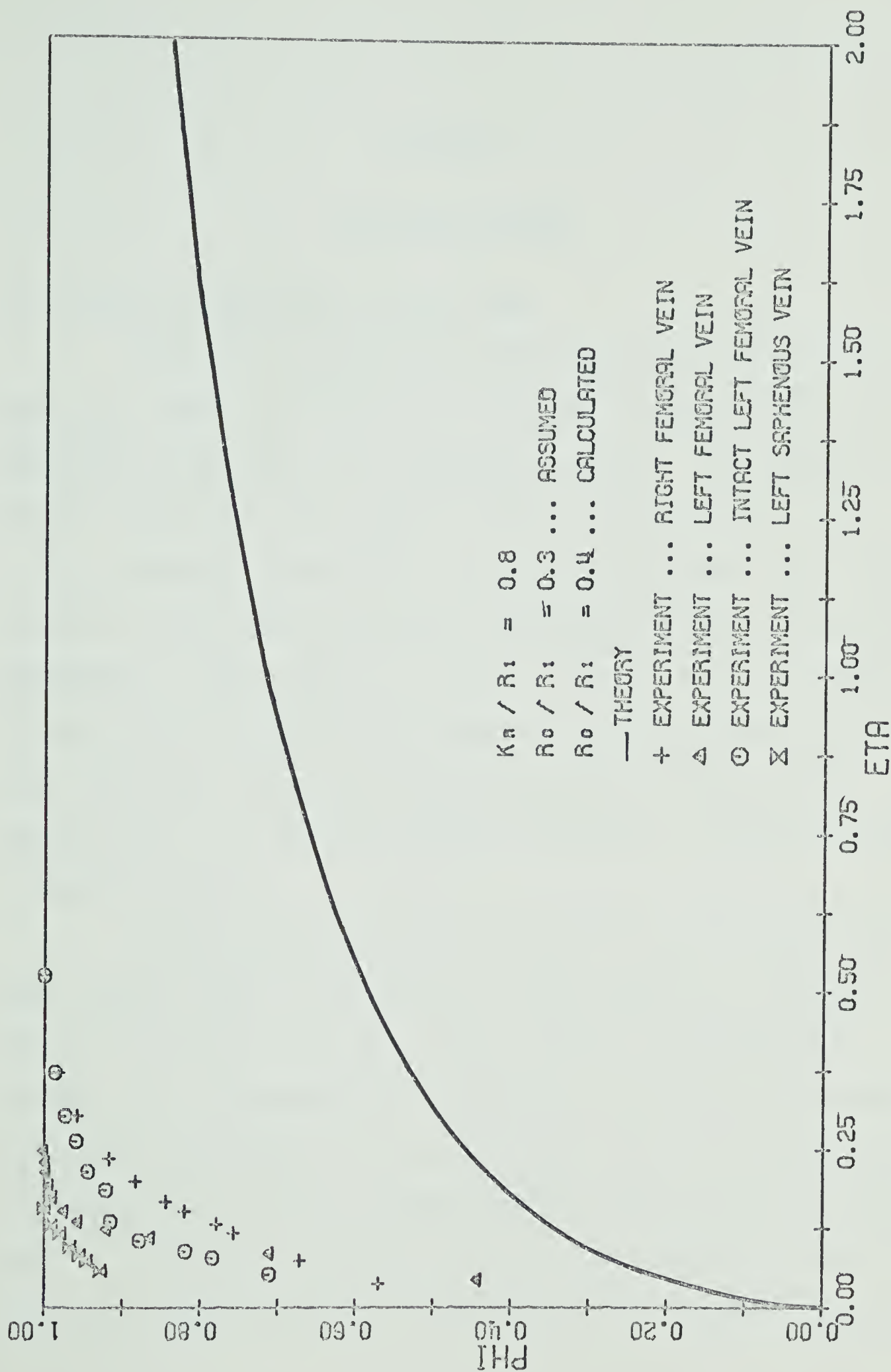


FIG. 4.8 PHYSIOLOGICAL EXPERIMENTS WITH THE CANINE VENOUS SYSTEM

CHAPTER V

CONCLUDING REMARKS

5.1 Scope and Limitation of Present Work

The dilatation equation has been derived for fairly general conditions: power law fluids; tube walls with a pressure-radius relationship representable by an exponential function, including both pressure limited and radius limited cases; and any dilatation ratio.

Agreement between theory and physical experiment suggest that the theory is applicable to physical situations which meet the principal requirements - namely $vt/R_1^2 \gg 1$, $L/R_1 \gg 1$ and no axial constraints. The most severe restriction would appear to be the requirement of no axial constraints. The requirement that $vt/R_1^2 \gg 1$, that is, the inertia terms in the equation of motion are absent, is readily controlled in physical systems.

In physiological systems, for blood of "mean viscosity" 0.1 poise, (Haynes [14]) in a capillary of 1 mm. diameter it is evident that this condition is met if $t > 1$ sec. That is, the dilatation equation for a locking material is applicable to any part of the vascular system with the exception of the great vessels and the microvasculature: the former because of their diameters and the latter because the continuum model is in question. Although the fluid is not initially at rest, the velocities in small veins are low enough that the dilatation equation can

be applied. However, its use is not limited to the cardiovascular system. Using the suitable exponential function for wall rheology it can also be used to study other systems, in particular the flow of axoplasm in nerve fibers (Johnson, Smith and Lock [17]) and latex in the laticifers of rubber trees (Frey-Wyssling [18]).

5.2 Suggestions for Further Work

Solution of the differential equations (22) and (31) with added time dependence would certainly be a noteworthy contribution and valuable extension of the work presented here. As well, the solution of an equation for elastic expulsion that included inertia effects would eliminate a major restriction of the equations developed here.

The results of the physiological experiments suggest that more extensive investigation be carried out. Better methods of determining pressure-radius relationships for blood vessels are warranted. Denervation could be used to isolate the vasoconstrictive element. Most important would be the development of a mathematical expression and physical model capable of simulating the effect of vasoconstriction. This, together with the addition of inertia terms and a pulsatile component, would provide an accurate model of the expulsion process in either the great veins or the large vessels of the arterial circulation; a truly significant contribution.

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APPENDIX A

PHYSICAL EXPERIMENTS

A.1 Calculation of Inner Radius

The thickness of the tube wall was determined by taking measurements with a micrometer at 25 randomly selected locations along a 36 inch length of tubing. The average thickness was found to be 0.0102 inches.

Having slit the tube open to make the previous measurements it was also possible to determine the tube's circumference, 0.79 inches, from which:

$$\text{Outer Diameter} = \frac{0.79}{\pi} = 0.250 \text{ inches}$$

and

$$\begin{aligned} \text{Inner Diameter} &= \text{O.D.} - 2 \times \text{Thickness} \\ &= 0.230 \text{ inches.} \end{aligned}$$

Despite the fact that the difference between the O.D. and I.D. was only 0.020 inches, it was decided to use the Inner Radius for all calculations as in the theory. Assuming that the cross-sectional area of the tube wall remains constant for dilatations as high as 20%, it can be determined from

$$\begin{aligned}
 A &= \pi(R_{\text{OUTER}}^2 - R_{\text{INNER}}^2) \\
 &= \pi\left[\left(\frac{0.250}{2}\right)^2 - \left(\frac{0.230}{2}\right)^2\right] \\
 &= 0.075 \text{ in.}^2
 \end{aligned}$$

Thus:

$$R_{\text{INNER}} = \left[\frac{\pi R_{\text{OUTER}}^2 - 0.075}{\pi} \right]^{1/2}$$

This expression was used to convert all measurements of R_{OUTER} to R_{INNER} .

A.2 Investigation of the Pressure vs Radius Relationship

The apparatus used in determining the Pressure vs Radius relationships for the Blue Latex tubes is shown in Figure A.1. The latex tubes were first placed inside a 24 inch length of copper tubing I.D. 0.31 inches, from the center portion of which an 8 inch test section had been milled leaving only 1/3 of the circumference. The copper tubing was then mounted in an upright position and the top displaced approximately 1/2 inch from vertical to ensure that the latex tube remained in contact with the copper surface. The pressure probe used here consisted of an 18 gauge hypodermic needle with beveled point honed down to leave a symmetrical truncated cone. A 22 gauge needle of sufficient length to allow the beveled end to clear the truncated cone was then inserted into the 18 gauge needle. The pair were then inserted into the latex tube,

the 22 gauge needle removed, and the probe cemented in place using a minute amount of Goodyear Pliobond Adhesive. The probe was positioned 2-1/4 inches above the lower end of the test section.

The pressure transducer was then connected and positioned to read the pressure at a point 5 inches above the lower end of the test section where a micrometer gauge was mounted to measure the increase in diameter of the latex tube, Figure A.2. The use of a 16X magnifying glass helped greatly in determining point of contact of latex tube and micrometer. Using the laboratory water supply, the system was pressurized and the tap adjusted to provide a minimal flow. Pressure was controlled by means of a finely adjustable screw clamp downstream from the copper tubing. Values of pressure and corresponding diameter were then recorded. A sample pressure recording is shown in Figure G.1. One hour was required for one complete cycle, that is from minimum to maximum pressure and back to minimum pressure. The yield point of the latex tubes was found to be 250 ± 5 mm Hg.

The procedure was repeated for 3 tubes, the results, all within experimental error (Appendix E), reduced to values of P vs R_{INNER} , and the resulting curve plotted (Figure A.3). The curve for decreasing pressure was chosen to represent the Pressure vs Radius relationship existing during the expulsion process. Note the permanent set of 0.0014 inches, from which $R_0 = 0.115 + 0.0014 = 0.1164$ inches.

Although the rheological characteristics of latex tubes are very dependent upon test conditions, especially time and temperature [6,7],

the fact that the expulsion process occurred in $0.1 < t < 20$ seconds did not appear to significantly affect the results.

A.3 Determination of K_P , M_P

As outlined in section 2.4.a, the relationship between pressure and radius for a yield material can be expressed as

$$P - P_0 = K_P \left[1 - e^{-\frac{M_P}{R_0} (R-R_0)} \right]$$

$$= K_P - K_P e^{-\frac{M_P}{R_0} (R-R_0)}$$

From which

$$\frac{\partial P}{\partial R} = \frac{K_P M_P}{R_0} e^{-\frac{M_P}{R_0} (R-R_0)}$$

$$= -\frac{M_P}{R_0} (P - P_0) + \frac{M_P K_P}{R_0}$$

and for $P_0 = 0$ the equation becomes

$$\frac{\partial P}{\partial R} = -\frac{M_P}{R_0} P + \frac{M_P K_P}{R_0}$$

The curve representing $P - P_0$ vs $R - R_0$ for decreasing pressure was differentiated graphically and the results plotted vs pressure (Figure A.4).

Thus the intercept at $P = 0$, $\frac{M_p K_p}{R_0} = 10,600$ mm.Hg./inch, and the slope $= - \frac{M_p}{R_0} = - 26.7$. Since $R_0 = 0.1164$, it follows that $M_p = 3.1$ and $K_p = 397$ mm.Hg.

Figure A.5 shows the relationship between measured values and those calculated from the representative exponential function.

A.4 Calculation of Phi and Eta

For a Newtonian fluid, the expression for η as given by equation (21), reduces to

$$\eta = \frac{2X}{R_0} \sqrt{\frac{\mu_0}{M_p K_p t}} .$$

Thus for any one set of test conditions the only variable is time. It is possible then to calculate a number of values of η for a range of t as determined by the Pressure vs Time recording. Values of pressure corresponding to the chosen values of time are then used to determine related radius values from the chosen exponential function, in this case

$$R - R_0 = \frac{R_0}{M_p} \ln\left(\frac{K_p}{K_p - P}\right) .$$

With this value of radius the value of ϕ is calculated from

$$\phi = \frac{R - R_0}{R_1 - R_0} .$$

Results of these calculations are recorded in Tables 1 to 21.

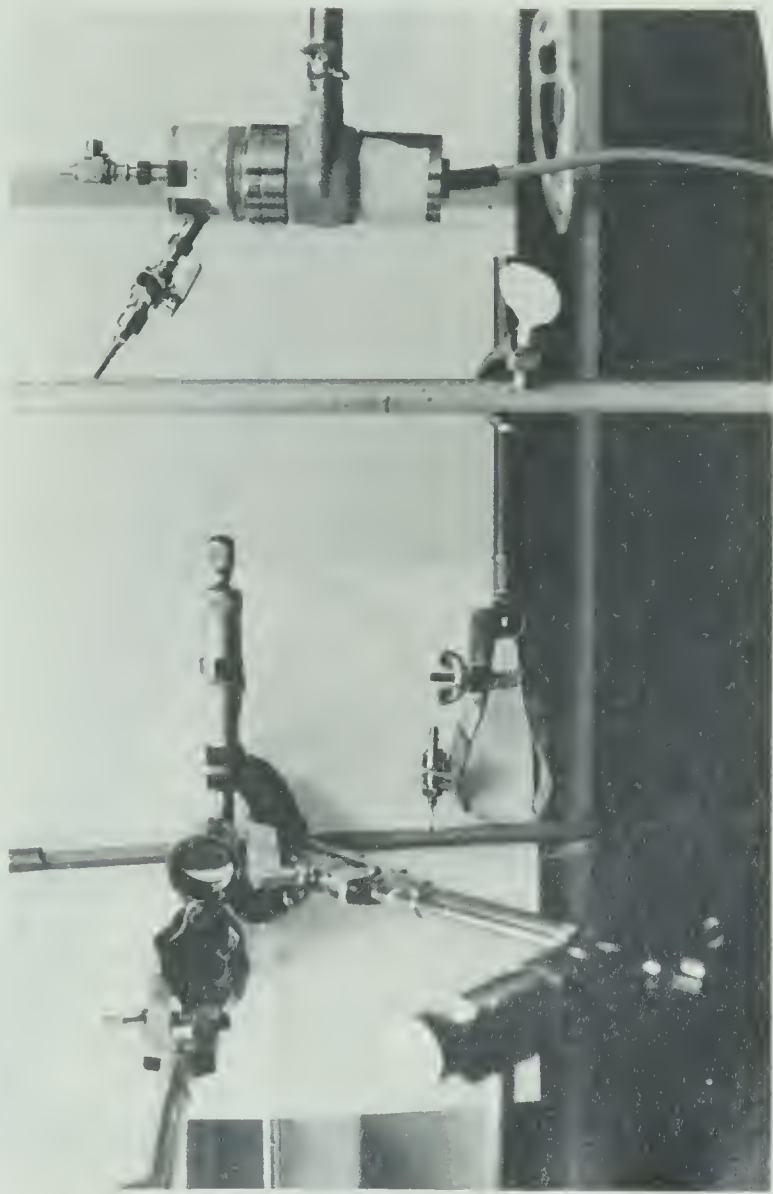


FIG. A.1 MEASUREMENT OF PRESSURE-RADIUS RELATIONSHIP



FIG. A.2 ENLARGEMENT OF TEST SECTION

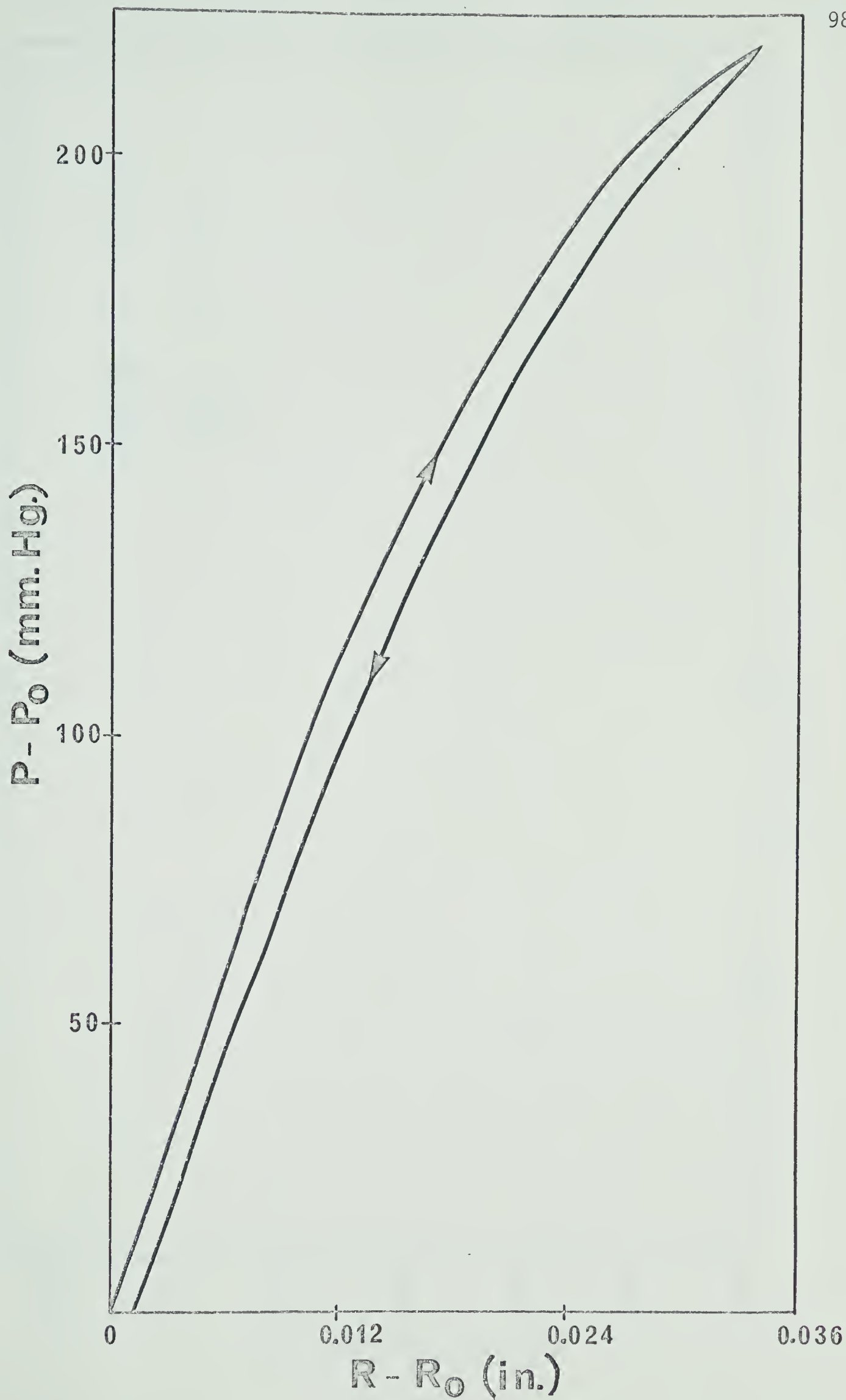


FIG. A.3 STATIC PRESSURE-RADIUS RELATIONSHIPS FOR BLUE LATEX TUBES

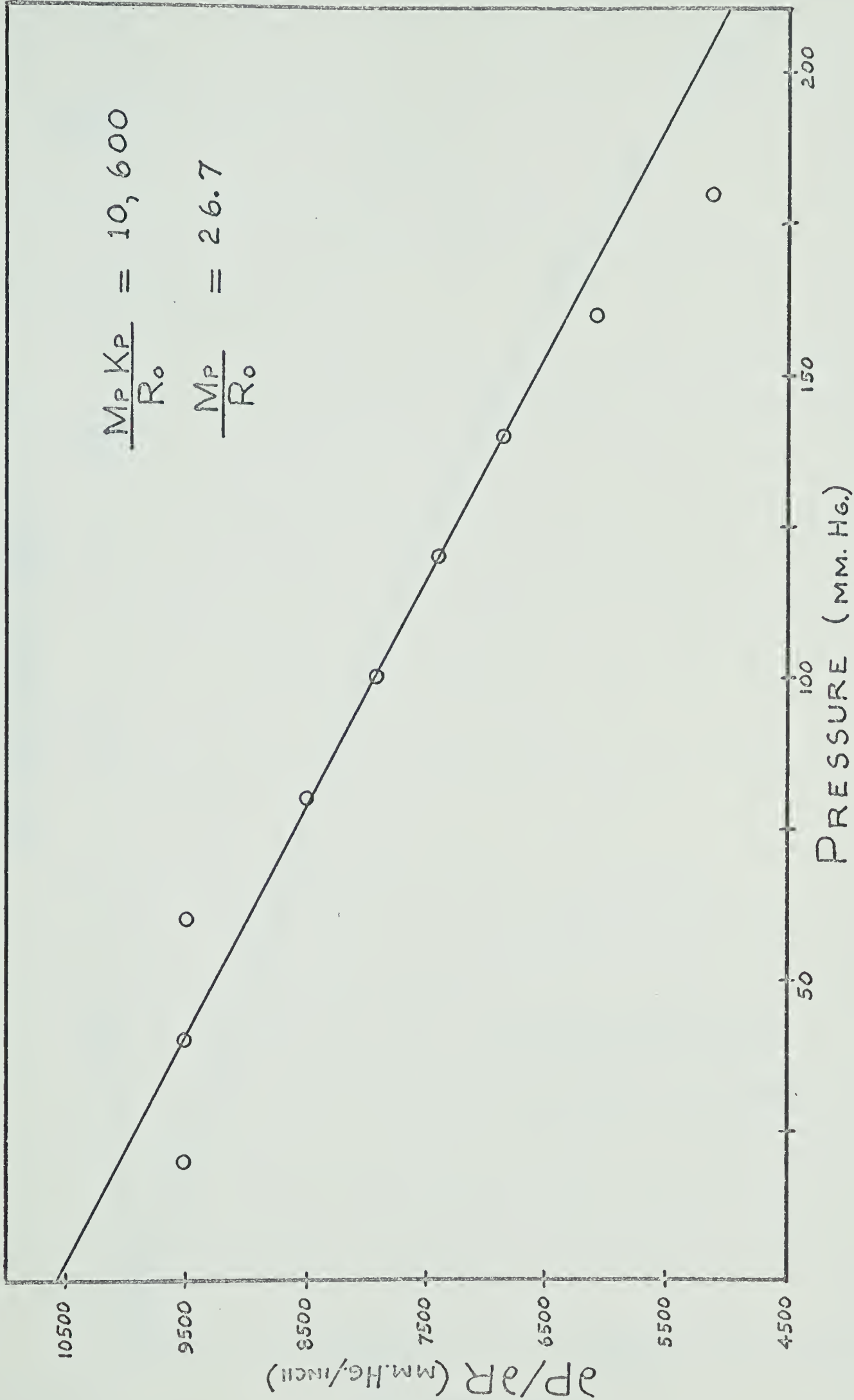


FIG. A.4 DETERMINATION OF M_P AND K_P

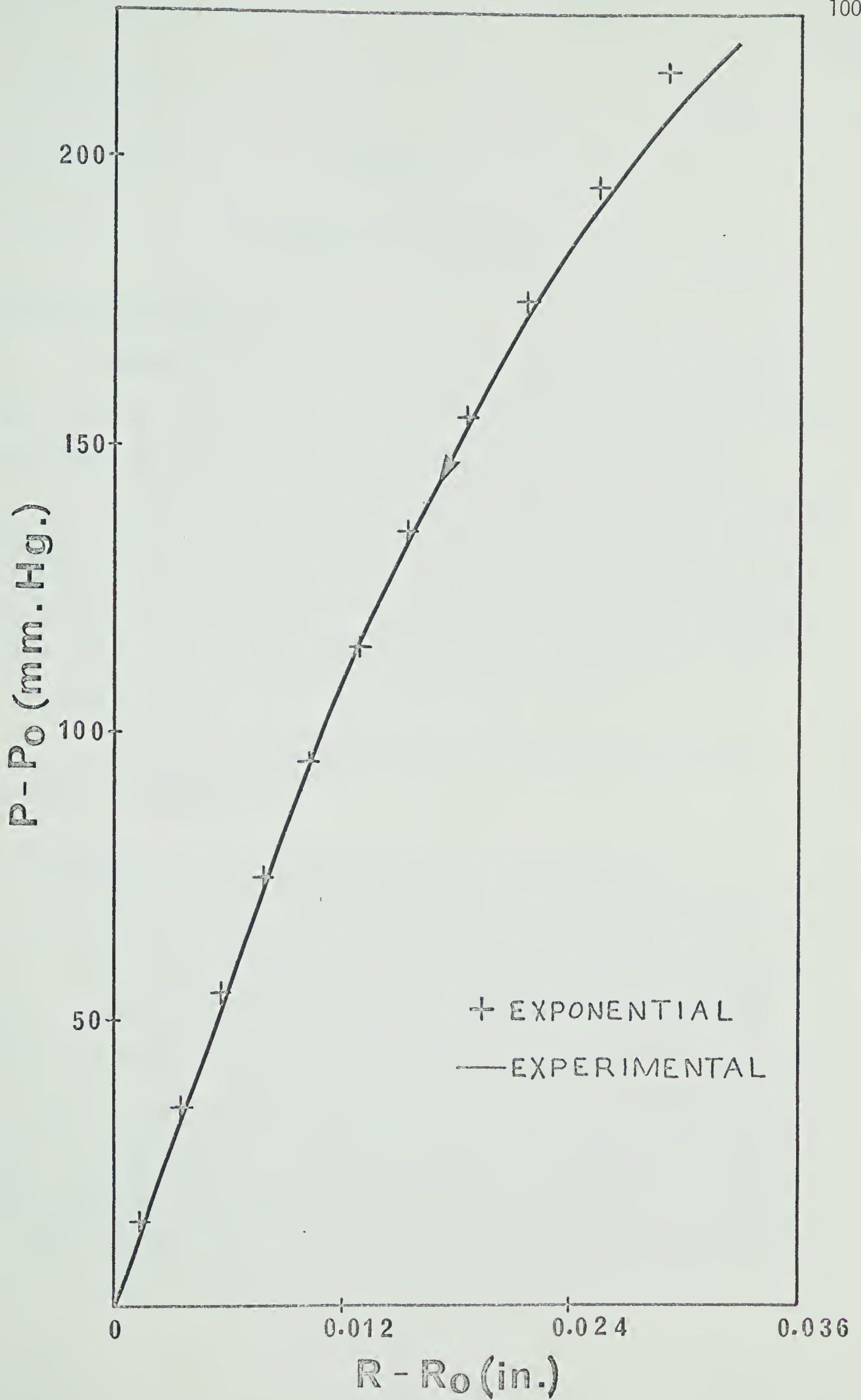


FIG. A.5 EXPONENTIAL FUNCTION FITTED TO ACTUAL CURVE

APPENDIX B

PHYSIOLOGICAL EXPERIMENTS

B.1 Calculation of Inner Radius

The small size of the veins used here, plus their collapsible nature, coupled with the difficulties encountered when working with body tissues made accurate measurement of the inner radius impossible.

The inner radius was chosen by combining the outer value of R_1 , measured from the intact vein, with an assumed dilatation ratio of 0.3. This value of R_0 was then used in the exponential function to calculate a value for R_1 . This calculated value came to 0.4 and allowed setting $\phi = 1.0$ for $R = R_1$.

The effect of assuming dilatation ratios of 0.2 and 0.4 is discussed in Appendix I.2.

B.2 Investigation of the Pressure vs Radius Relationships

Due to limited time and assistance, it was impossible to conduct experiments to determine accurate Pressure vs Radius relationships on veins extracted from the dogs. Literature displaying good qualitative information is limited. However, in keeping with the terms of reference, good qualitative information was obtained, Alexander [3]. Figure 3 on page 1083 of this reference gives information about both volume and radial displacement as a function of pressure for a segment of a dog's jugular vein. This information was used to construct Figure

B.1 and hence determine R_0 which was then used in constructing the Pressure vs Radius graph of Figure B.2.

B.3 Determination of K_R , M_R

Equation (29) can be differentiated with respect to P and rearranged in the form

$$\frac{\partial R}{\partial P} = - \frac{M_R}{P_0} (R - R_0) + \frac{M_R K_R}{P_0} .$$

Values for $\partial R / \partial P$ were obtained from graphical differentiation of Figure B.2 and plotted vs $R - R_0$ as shown in Figure B.3, from which:

$$\frac{M_R K_R}{P_0} = \text{Intercept on } \frac{\partial R}{\partial P} \text{ Axis} = 0.056$$

$$- \frac{M_R}{P_0} = - \text{Slope} = - 0.393$$

Thus $K_R = 0.056 \times P_0 / M_R = 0.143$ inches and assuming $P_0 = 760$ mm.Hg.
 $M_R = 299$.

However, these values are only for a vein of the size of the jugular vein. For the physiologic conditions under which the tests were conducted it is reasonable to expect that the main resistance to dilatation was provided by the elastin fibers, with few collagen fibers taking part. Thus although the slope of the graph, Figure B.3 remains the same for any size of vein, its position would shift. For the

smaller veins used here, the line would be shifted to the left and hence K_R would also be less.

For the experiments K_R was determined as follows: under physiologic conditions, the increase in pressure required to stretch the elastin fibres is very small. Therefore $P_1 \approx P_0$. Hence $R_1 - R_0 \approx K_R$. For the right femoral vein then, assuming a dilatation ratio of 0.3, $R_0 = 0.3 \times 0.0591 = 0.0177$ inches. It follows that $K_R = 0.0591 - 0.0177 = 0.0414$. Similar calculations were carried out for the other tests and are recorded in Tables 22 to 25. For all tests, the value of K_R/R_1 remained at ≈ 0.8 .

B.4 Calculation of ϕ and η

A similar pattern to that outlined in A.4 was followed. In this case

$$\eta = \frac{2X}{R_0} \sqrt{\frac{M_R \mu_0}{P_0 t}}$$

where $\mu_0 = 0.06$ poise [14,15]. $R - R_0$ was determined from the equation describing Pressure vs Radius relationships for a locking material:

$$R - R_0 = K_R \left[1 - e^{-\frac{M_R}{P_0} (P - P_0)} \right].$$

The above equation was also used to calculate R_1 after which

$$\phi = \frac{R - R_0}{R_1 - R_0}$$

The results are recorded in Tables 22 to 25.

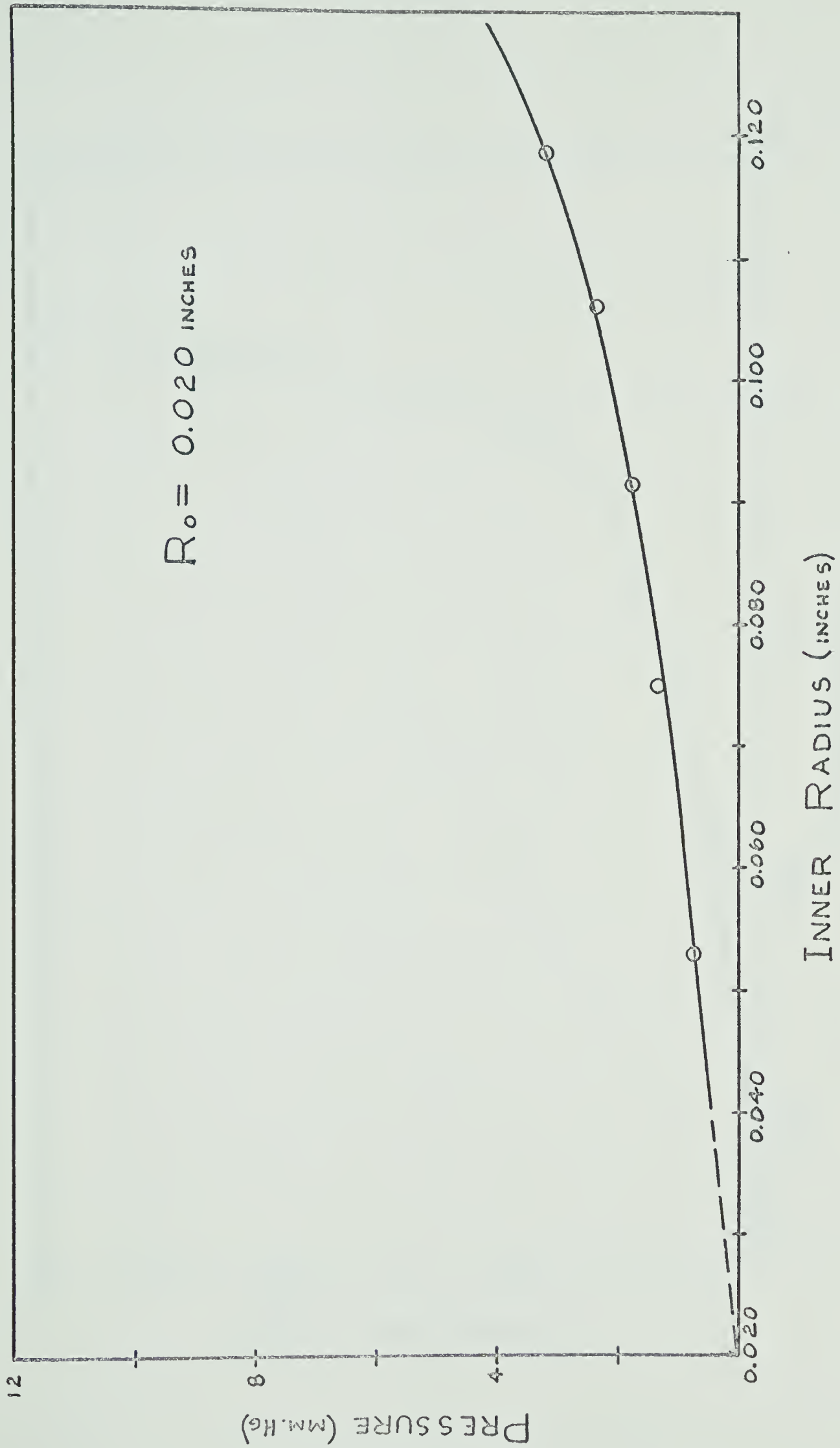


FIG. B.1 DETERMINATION OF R_o

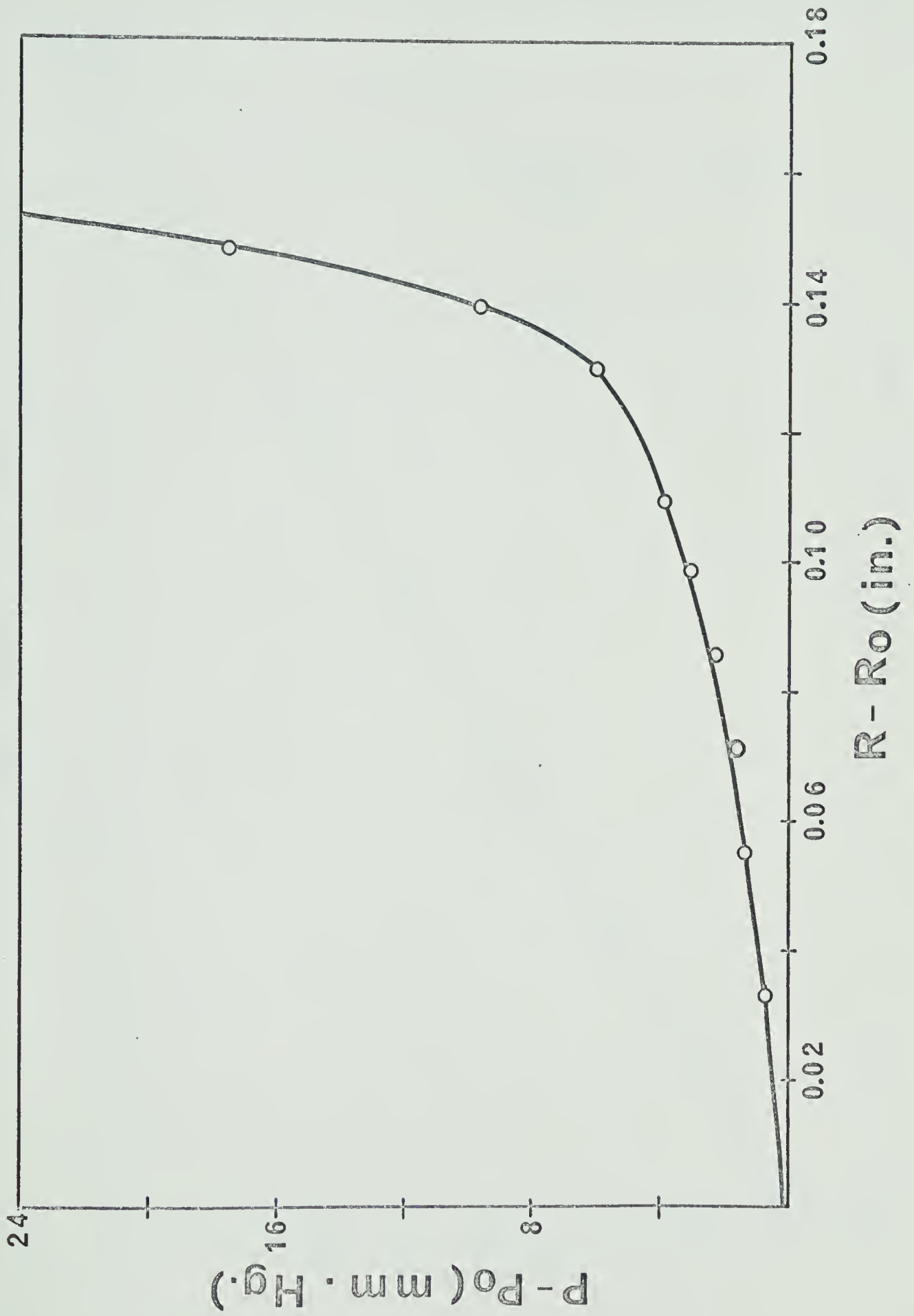


FIG. B.2 STATIC PRESSURE-RADIUS RELATIONSHIP FOR A CANINE JUGULAR VEIN

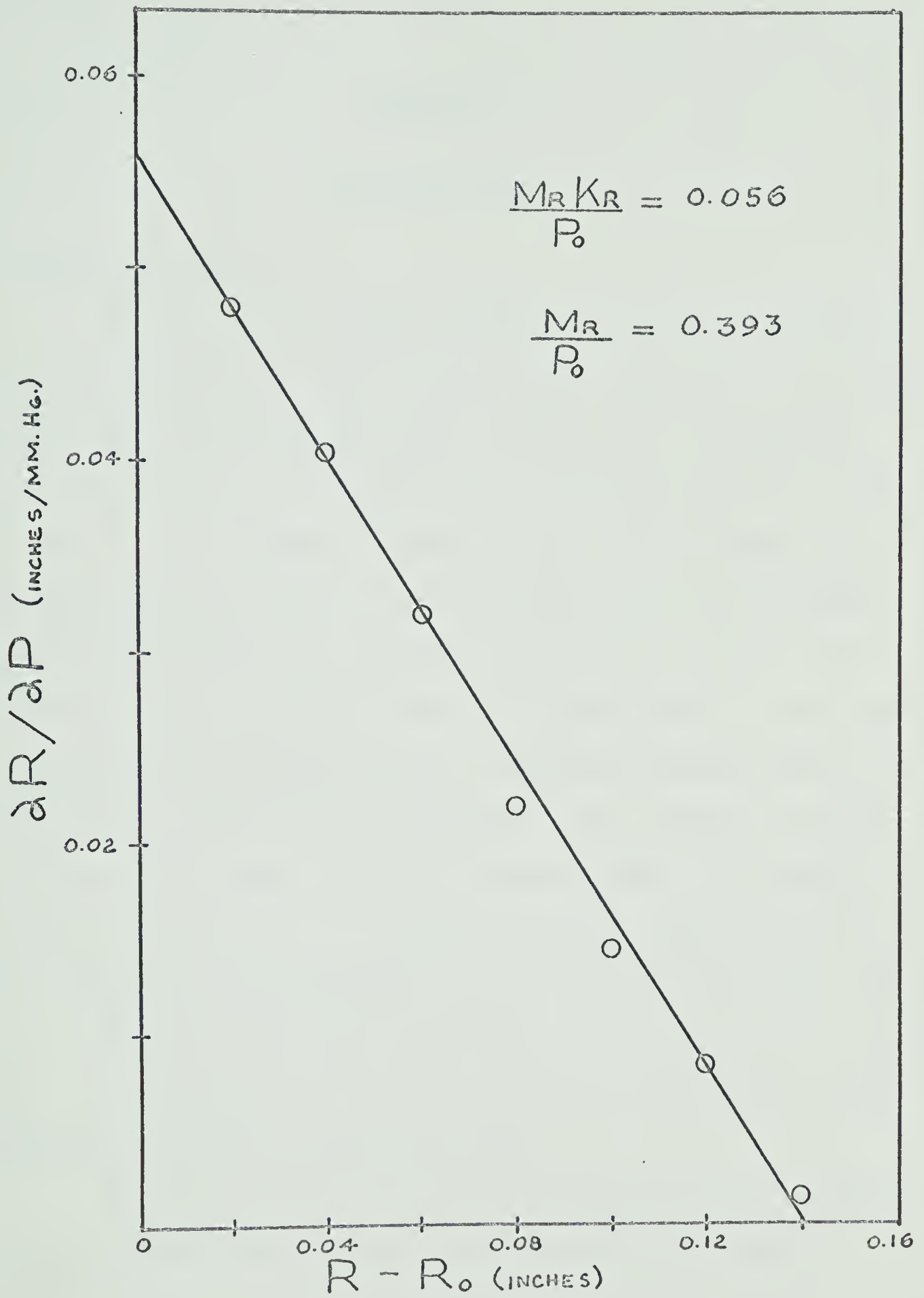


FIG. B.3 DETERMINATION OF M_R , K_R

APPENDIX C

INSTRUMENTATION

C.1 Recording System

A Hewlett Packard Sanborn 7700 system was used and contained the following components: 350-1500A Low Level Preamplifier with 350-2B plug in, used in producing the signal marking $t = 0$, the 350-1100C Carrier Preamplifier used in conjunction with the pressure transducers, and the 7714A Oscillographic Recording System, used to produce the tracings of pressure vs time. Specifications indicate the 350-1500A Low Level Preamplifier has a bandwidth of 0 to 90 Hz. down 3db. The amplifier rise time is 25 milliseconds for 99.9% response to step input or 15 milliseconds for 99% response. The 350-1100C preamplifier receiving signals from the pressure transducer has a frequency response down 3db at 480 Hz. and a rise time of approximately 1 millisecond. The frequency response of the Oscillographic Recording System including the Driver Amplifiers and Galvonometers is 0 to 125 cycles per second within 3db at 10 divisions peak-to-peak amplitude at the center of the recording chart. The response time is 5 milliseconds for a 10 division displacement, 10% to 90% with 4% or less overshoot.

Thus the equipment is capable of producing an accurate record of events for which the time scale is greater than 0.01 seconds.

C.2 Pressure Transducer

The frequency response of a pressure transducer depends upon connecting hardware, catheter length and diameter, pressure medium, and its internal volume and volume displacement. It can be estimated using the expression:

$$f_n = \frac{D}{8} \sqrt{\frac{3(\frac{\Delta P}{\Delta V})}{\pi \rho L}}$$

where f_n = natural frequency
 D = diameter of the catheter
 L = length of the catheter
 ΔP = pressure change
 ΔV = volume change
 ρ = liquid density

For the physical experiments utilizing the Hewlett Packard, model 267AC transducer, and Bardic-Deseret Angiocath

$$D = 1.47 \text{ mm.}$$

$$L = 50.8 \text{ mm.}$$

$$\Delta P = 100 \text{ mm.Hg.}$$

$$\Delta V = 0.03 \text{ mm.}^3$$

$$\rho = 1.23 \text{ gms/cc}$$

and hence $f_n = 479 \text{ Hz.}$

Similarly, for the Hewlett Packard transducer, model 268B, connecting tubing and catheter used in the physiological experiments,

$$D = 2.08 \text{ mm.}$$

$$L = 203 \text{ mm.}$$

$$\Delta P = 100 \text{ mm.Hg.}$$

$$\Delta V = 0.3 \text{ mm.}^3$$

$$\rho = 1.06 \text{ gm/cc}$$

resulting in $f_n = 119 \text{ Hz.}$

Since observed time scales were greater than 0.01 seconds, it is clear that both recorder and transducers were suitable.

Both recording system and pressure transducers were purchased from Hewlett Packard (Canada) Ltd., 11745 Jasper Avenue, Edmonton, Alberta.

APPENDIX D

MEASUREMENT OF VISCOSITY

The viscosity of the glycerine - water solutions was measured with a FANN VISCOMETER, Model 35. The viscometer was equipped with the R₁ rotor, B₁ bob and F₁ spring.

$$\text{Viscosity (Poise)} = \frac{5.077 \times F \times \theta}{\text{Shear Rate Range}}$$

RPM	3	100	200	300	600
Shear Rate Range	5.112	170.4	340.8	511.2	1,022

F = spring factor = 1
θ = dial deflection (degrees)

The viscosity of the mixture for each series of tests was determined by measuring the dial deflection at both a high and low shear rate and taking the average value.

The results are listed below.

Tests I - II	Viscosity
Deflection at 100 RPM 143°	4.26 poise
Deflection at 200 RPM 286°	4.26 poise

$$\mu_0 = \frac{4.26 + 4.26}{2} = 4.26 \text{ poise}$$

Tests V - VII

Viscosity

Deflection at 6 RPM 21.75°

10.90 poise

Deflection at 3 RPM 11.0°

10.70 poise

$$\mu_0 = \frac{10.9 + 10.7}{2} = 10.80 \text{ poise}$$

Tests VIII - IX

Viscosity

Deflection at 6 RPM 13.5°

6.70 poise

Deflection at 100 RPM 224°

6.66 poise

$$\mu_0 = \frac{6.70 + 6.66}{2} = 6.68 \text{ poise}$$

APPENDIX E
FORTRAN SOURCE PROGRAM FOR DETERMINING
PHI AND ETA

E.1 Solving for PHI and ETA - Yielding Material

```

C   PPRIME REPRESENTS THE DIFFERENTIAL OF PHI WITH RESPECT TO ETA
C   EINTGL REPRESENTS THE FUNCTION F(ETA)
C   FIOETA REPRESENTS THE INTEGRAL OF EXP(F(ETA))
C   FIOINF REPRESENTS THE INTEGRAL OF EXP(F(ETA)) FROM ZERO TO INFINITY
C   EETA REPRESENTS E(ETA)

      DIMENSION PHI(2,1005), PPRIME(2,1005), ETA(1005), EINTGL(1005),
1    FIOETA(1005),EETA(1005),ZETA(1005)

C
C   POINTS REPRESENTING VALUES WITHIN THE FUNCTION F(ETA)
C   THESE THREE POINTS ARE CALLED BY THE NUMERICAL INTEGRATION PROCESS
C   WHICH UTILIZES A 3-POINT SIMPSON'S RULE
C   THE FUNCTIONS LISTED BELOW APPLY ONLY TO THE PRESSURE LIMITED CASE
C
      PE1(E)=( 2.0*EXP((WR/FR)*B*FI1)*(DR**C)*(SLOPE1**F)*E)/((DR+A*FI1)
1    **C)-(WR*(1.0+B*FI1)*SLOPE1)/(DR/A+FI1)
      PE2(E)=( 2.0*EXP((WR/FR)*B*FI2)*(DR**C)*(SLOPE2**F)*E)/((DR+A*FI2)
1    **C)-(WR*(1.0+B*FI2)*SLOPE2)/(DR/A+FI2)
      PE3(E)=( 2.0*EXP((WR/FR)*B*FI3)*(DR**C)*(SLOPE3**F)*E)/((DR+A*FI3)
1    **C)-(WR*(1.0+B*FI3)*SLOPE3)/(DR/A+FI3)

C
C
C   SETTING THE FLUID RHEOLOGY PARAMETER TO CORRESPOND TO A NEWTONIAN FLUID
C
      FR = 1.0
      DETA = 0.04
      INF = 802
      INFP = INF + 1
      INFM = INF - 1

C
C   DO LOOP CALCULATING VALUES OF PHI(ETA) FOR A PARTICULAR DILATATION
C   RATIO, DR
C
      DO 70 J = 1,5
      NEW = 1
      READ (5,61) DR
61  FORMAT (6X,F3.1)
      A = 1.0-DR
      B = (1.0/DR) - 1.0
      C = (2.0*FR+1.0)/FR
      D = (3.0*FR+2.0)
      F = (FR-1.0)/FR
      G = 1.0/DR
      H = 3.0*FR+1.0

```



```

C
C   CALCULATING VALUES OF ETA
C
      CO 3 L = 2,INF
      ETA(L) = (L-2)*DETA
      ZETA(L) = ETA(L)*(DR**1.5)
3     CONTINUE
      IC = 1
      K = 1
      PHI(1,2) = 0.0
C
C   CALCULATION OF PHI(ETA) FOR DR = 1.0
C
      DO 5 L = 3,102
      PHI(K,L) = ERF(ETA(L))
5     CONTINUE
      WRITE (6,100) ((PHI(K,L),ETA(L)), L = 2,102)
C
C   FIRST APPROXIMATION ...LOWER BOUND SOLUTION FOR A NEWTONIAN FLUID
C
      DO 28 L = 3,INF
      PHI(K,1) = (((DR**D+(1.0=(DR**D))*ERF(ZETA(L)))*G)-DR)/A
28    CONTINUE
C
C   DO LOOP CALCULATING PHI(ETA) FOR A PARTICULAR WALL RHEOLOGY PARA-
C   METER, WR
C
      DO 60 M = 1,6
      READ (5,71) WR
71    FORMAT(6X,F7.4)
      IF (NEW.EQ.1) GO TO 36
      DO 35 L = 2,INF
      PHI(K,L) = PHI(K+1,L)
35    CONTINUE
      GO TO 36
18    IC = IC + 1
36    PHI(K,INF + 1) = PHI(K,INF)
      IF (NEW.EQ.2) GO TO 91
      PHI(K,1) = -PHI(K,3)
      PHI(K,INF+2) = PHI(K,INFP)
C
C   DERIVATIVE OF PHI WITH RESPECT TO ETA, FIRST ITERATION ONLY
C
      JUMP = 1
      DO 13 L = 2,INFP
      IF (JUMP.EQ.2) GO TO 89
      PPRIME (K,L) = (PHI(K,L+1)-PHI(K,L-1))/(2.0*DETA)
      IF (PPRIME(K,L).LE.1.E-40) GO TO 89

```



```

      GO TO 13
89  PPRIME(K,1) = 0.0
      JUMP = 2
13  CONTINUE
90  NOMR = L-1
      NOTM = L
      GO TO 93
91  CONTINUE
C
C  DERIVATIVE OF PHI WITH RESPECT TO ETA, SUCCEEDING ITERATIONS
C
      N1 = 1
      MUCH = 1
      DO 92 L = 2,INFP
      IF (MUCH.EQ.2) GO TO 77
      N1 = N1+1
      IF (ABS(EINTGL(N1)).GT.(174.0)) GO TO 94
      OVER = EXP(EINTGL(N1))*D
      OVERM = OVER*FIOINF*(((DR/A)+PHI(K,1))*H)
      IF (OVERM.GE.1.E+40) GO TO 77
      PPRIME(K,L) = (1.0-DR**D)/((A**D)*OVERM)
      GO TO 92
77  PPRIME(K,1) = 0.0
      MUCH = 2
92  CONTINUE
94  NOMR = L - 1
      NOTM = NOMR + 1
93  CONTINUE
C
C  CALCULATION OF THE FUNCTION F(ETA)
C
      EINTGL(2) = 0.0
      DO 82 L = 3,INF
      IF (L.EQ.3) GO TO 80
      P1 = ETA(L-2)
      P2 = ETA(L-1)
      P3 = ETA(L)
      FI1 = PHI(K,L-2)
      FI2 = PHI(K,L-1)
      FI3 = PHI(K,L)
      SLOPE1 = PPRIME(K,L-2)
      SLOPE2 = PPRIME(K,L-1)
      SLOPE3 = PPRIME(K,L)
      IF (SLOPE3.LT.1.E-40) GO TO 88
      ELEMNT = DETA*(PE1(P1) + 4.0*PE2(P2) + PE3(PE))/3.0
      EINTGL(L) = EINTGL(L-2) + ELEMNT
      IF (ABS(EINTGL(L)).GT.174.0) GO TO 88
      GO TO 82
80  P1 = ETA(L-1)

```



```

      P2 = ETA(L)
      FI1 = PHI(K,L-1)
      FI2 = PHI(K,L)
      SLOPE1 = PPRIME(K,L-1)
      SLOPE2 = PPRIME(K,L)
      ELEMNT = DETA*(PE1(P1) + PE2(P2))/2.0
21  EINTGL(L-1) + ELEMNT
82  CONTINUE
      GO TO 24
88  N OMR = L-1
      IF (NOMR.GE.INFM)GO TO 24
      DO 31 L = NOTM,INF
      EINTGL(L) = EINTGL(L-1)
31  CONTINUE
24  CONTINUE
C
C  CALCULATION OF THE INTEGRAL OF EXP(-F(ETA))
C
      FIOETA(2) = 0.0
      N1 = 1
      DO 83 L = 3,NOMR
      NOM = L
      NOMM = NOM+1
      IF (L.EQ.3) GO TO 81
      N1 = N1 + 1
      N2 = N1 + 1
      N3 = N2 + 1
      ELEMNT = DETA*(1.0/EXP(EINTGL(N1))+4.0/EXP(EINTGL(N2))+1.0/EXP(
1  EINTGL(N3)))/3.0
      FIOETA(L) = FIOETA(L-2) + ELEMNT
      FIOINF = FIOETA(L)
      STP = ABS (FIOETA(L) - FIOETA(L-1))
      IF (STP.LT.1.E-06) GO TO 99
99  CONTINUE
      GO TO 83
81  N1 = N1 + 1
      N2 = N1 + 1
      ELEMNT = DETA*(EXP(EINTGL(N1))+EXP(EINTGL(N2)))/2.0
      FIOETA(L) = FIOETA(L-1) + ELEMNT
      FIOINF = FIOETA(L)
      N1 = 1
83  CONTINUE
      COEF = ((1.0/(1.0-DR))**(3.0*FR+2.0))-((DR/(1.0-DR))**(3.0*FR+2.0)
1)
C
C  CALCULATION OF E(ETA)
C
      DO 15 L = 3,NOM

```



```

      EETA(L) = COEF*FIOETA(L)/FIOINF
15  CONTINUE
      PHI(K+1,2) = 0.0
      SUB = DR/A
      STOUT = (DR/A)**D
C
C  CALCULATION OF PHI(ETA)
C
      DO 22 L = 3,NOM
      SUM = (EETA(L) + STOUT)**G
      PHI(K+1,L) = SUM-SUB
22  CONTINUE
      NEW = 2
C
C  COMPARISON OF NEW VALUE OF PHI*ETA) WITH THAT OF PREVIOUS ITERATION
C
      L = 2
12  DIF = ABS(PHI(K+1,L) - PHI(K,L))
      IF ((DIF).LE.(1.E-05)) GO TO 16
17  DO 19 L = 2,NOM
      PHI(K,L) = PHI(K+1,L)
19  CONTINUE
      IF (NOM.GE.INFM) GO TO 30
      DO 29 L = NOMM,INF
      PHI(K,L) = 1.0
29  CONTINUE
30  PHI(K,INF) = 1.0
      GO TO 18
16  L = L + 1
      IF ((L).FT.(NOM)) GO TO 20
      GO TO 12
20  CONTINUE
      IF (NOM.GE.INFM) GO TO 34
      DO 23 L = NOMM,INF
      PHI(K+1,L) = 1.0
23  CONTINUE
34  CONTINUE
C
C  CALCULATION OF DERIVATIVE OF PHI WITH RESPECT TO ETA, EVALUATED AT
      ETA = 0
C  THIS IS ANALOGOUS TO THE EXPULSION RATE AT X = 0
C
      PPMO = (1.0-DR**D)/((A**D)*D*(SUB**H)*FIOINF)
      PPMOLG = ALOG10(PPMO)
      WRITE (6,55) FR
55  FORMAT(1H1,'THE VALUE OF N FOR THIS RUN IS .....',1X,F3.1)
      WRITE (6,57) WR
57  FORMAT(1H0,'THE VALUE OF M FOR THIS RUN IS ....',F7.4)
      WRITE (6,56) DR

```



```
56  FORMAT (1H0,'THE VALUE OF RO/R1 FOR THIS RUN IS .....',F3.1)
    WRITE (6,59) PPMO
59  FORMAT (1H0, 'THE VALUE OF PPMO IS .....', F8.3)
    WRITE (6,58) PPMOLG
58  FORMAT (1H0, 'THE VALUE OF LOG10 PPMO IS .....', F6.3)
    WRITE (6,25) IC
25  FORMAT (1H0, 'NUMBER OF ITERATIONS .....',I5)
    WRITE (6,100) ((PHI(K+1,L), ETA(L)),L = 2,102)
100 FORMAT (6(F6.3,F6.3))
60  CONTINUE
70  CONTINUE
    STOP
    END
```


E.2 Solving for PHI and ETA - Locking Material

Represented here are the particular functions which must be substituted into the program listed in Appendix E.1.

$$PE1(E) = (2.0*(DR**2.0)*((PAR-A*FI1)**(1.0/FR))*(SLOPE1**F)*E)/$$

$$1 ((DR+A*FI1)**C)+(((DR+A*FI1)/(PAR-A*FI1))*SLOPE1)/(DR/A+FI1)$$

$$PE2(E) = (2.0*(DR**2.0)*((PAR-A*FI2)**(1.0/FR))*(SLOPE2**F)*E)/$$

$$1 ((DR+A*FI2)**C)+(((DR+A*FI2)/(PAR-A*FI2))*SLOPE2)/(DR/A+FI2)$$

$$PE3(E) = (2.0*(DR**2.0)*((PAR-A*FI3)**(1.0/FR))*(SLOPE3**F)*E)/$$

$$1 ((DR+A*FI3)**C)+(((DR+A*FI3)/(PAR-A*FI3))*SLOPE3)/(DR/A+FI3)$$

APPENDIX F
TABLES OF RESULTS

TABLE 1

PHYSICAL EXPERIMENT TEST I-5

$X = 5$ inches

$$\frac{R_0}{R_1} = 0.79$$

$\mu_0 = 4.26$ poise

Time Seconds	η	$P-P_0$ mm.Hg.	$\phi(\eta)$
0.00	∞	225	1.000
0.01	1.385	222	0.979
0.02	0.979	211	0.906
0.03	0.800	200	0.838
0.04	0.692	189	0.773
0.05	0.619	178	0.711
0.06	0.565	168	0.658
0.07	0.523	160	0.617
0.08	0.490	154	0.587
0.09	0.462	148	0.558
0.10	0.438	141	0.525
0.12	0.400	131	0.479
0.14	0.370	121	0.435
0.16	0.346	112	0.396
0.20	0.310	94	0.323
0.24	0.283	78	0.262
0.30	0.253	56	0.182
0.40	0.219	30	0.094
0.50	0.196	13	0.040
0.70	0.166	2	0.006
0.90	0.146	1	0.003
1.00	0.138	0	0.000

TABLE 2

PHYSICAL EXPERIMENT TEST I-15

$X = 15$ inches $\frac{R_0}{R_1} = 0.79$
 $\mu_0 = 4.26$ poise

Time Seconds	η	$P-P_0$ mm. Hg.	$\phi(\eta)$
0.00	∞	225	1.000
0.07	1.570	225	1.000
0.08	1.469	223	0.986
0.09	1.385	222	0.979
0.10	1.314	221	0.973
0.11	1.253	220	0.966
0.12	1.199	218	0.952
0.13	1.152	215	0.932
0.14	1.110	214	0.926
0.15	1.073	212	0.913
0.16	1.039	209	0.894
0.18	0.979	205	0.868
0.20	0.929	201	0.844
0.22	0.886	198	0.826
0.24	0.848	194	0.802
0.26	0.815	190	0.779
0.30	0.759	183	0.739
0.40	0.657	165	0.642
0.50	0.588	147	0.553
0.60	0.536	131	0.479
0.70	0.497	115	0.409
0.80	0.465	100	0.347
0.90	0.438	85	0.288
1.00	0.415	73	0.243
1.25	0.372	46	0.147
1.50	0.339	27	0.084
2.00	0.294	6	0.018
2.20	0.280	2	0.006
3.00	0.240	0	0.000

TABLE 3

PHYSICAL EXPERIMENT TEST I-25

$X = 25$ inches $\frac{R_0}{R_1} = 0.79$
 $\mu_0 = 4.26$ poise

Time Seconds	η	$P-P_0$ mm.Hg.	ϕ
0.00	∞	225	1.000
0.17	1.680	225	1.000
0.18	1.632	224	0.993
0.20	1.548	223	0.986
0.21	1.511	222	0.979
0.25	1.385	220	0.966
0.30	1.264	216	0.939
0.35	1.171	213	0.919
0.40	1.095	210	0.900
0.45	1.032	205	0.868
0.50	0.979	202	0.850
0.60	0.894	194	0.802
0.70	0.828	185	0.750
0.80	0.774	178	0.711
0.90	0.730	170	0.668
1.00	0.692	164	0.637
1.25	0.619	144	0.539
1.50	0.565	125	0.452
1.75	0.523	107	0.375
2.00	0.490	90	0.307
2.50	0.438	64	0.210
3.00	0.400	44	0.140
3.50	0.370	29	0.091
4.00	0.346	20	0.062
5.00	0.310	10	0.030
6.00	0.283	5	0.015
7.00	0.262	4	0.012
8.00	0.245	2	0.006

TABLE 4

PHYSICAL EXPERIMENT TEST II-5

$X = 5$ inches $\frac{R_0}{R_1} = 0.92$
 $\mu_0 = 4.26$ poise

Time Seconds	η	$P-P_0$ mm.Hg.	$\phi(\eta)$
0.00	∞	100	1.000
0.01	1.385	100	1.000
0.02	0.979	98	0.977
0.03	0.800	97	0.965
0.04	0.692	95	0.942
0.05	0.619	90	0.886
0.06	0.565	85	0.830
0.07	0.523	81	0.786
0.08	0.490	75	0.722
0.09	0.462	70	0.668
0.10	0.438	66	0.627
0.11	0.418	61	0.575
0.13	0.384	52	0.484
0.15	0.358	44	0.405
0.18	0.326	34	0.309
0.22	0.295	22	0.196
0.26	0.272	14	0.124
0.30	0.253	8	0.070
0.40	0.219	1	0.009
0.50	0.196	0	0.000

TABLE 5
PHYSICAL EXPERIMENT TEST II-15
X = 15 inches
 $\mu_0 = 4.26$ poise
 $\frac{R_0}{R_1} = 0.92$

Time Seconds	η	P-P ₀	$\phi(\eta)$
0.00	∞	100	1.000
0.03	2.399	100	1.000
0.04	2.077	99	0.988
0.05	1.858	98	0.977
0.06	1.696	98	0.977
0.07	1.570	97	0.965
0.08	1.469	97	0.965
0.09	1.385	96	0.954
0.10	1.314	95	0.942
0.11	1.253	94	0.931
0.12	1.199	93	0.920
0.13	1.152	92	0.908
0.14	1.110	90	0.886
0.16	1.039	86	0.841
0.18	0.979	83	0.808
0.20	0.929	80	0.775
0.22	0.886	75	0.722
0.24	0.848	72	0.690
0.26	0.815	69	0.658
0.30	0.759	61	0.575
0.35	0.702	53	0.494
0.40	0.657	45	0.415
0.45	0.619	39	0.356
0.50	0.588	34	0.309
0.60	0.536	24	0.215
0.75	0.480	14	0.124
1.00	0.415	3	0.026
1.25	0.372	0	0.000

TABLE 6
PHYSICAL EXPERIMENT TEST II-25
 $X = 25$ inches
 $\mu_0 = 4.26$ poise
 $\frac{R_0}{R_1} = 0.92$

Time Seconds	η	$P-P_0$ mm.Hg.	$\phi(\eta)$
0.00	∞	100	1.000
0.13	1.921	100	1.000
0.14	1.851	99	0.988
0.16	1.731	98	0.977
0.18	1.632	96	0.954
0.20	1.548	95	0.942
0.22	1.476	94	0.931
0.24	1.414	92	0.908
0.26	1.358	90	0.886
0.30	1.264	87	0.852
0.35	1.171	82	0.797
0.40	1.095	77	0.743
0.45	1.032	73	0.700
0.50	0.979	68	0.647
0.60	0.894	60	0.565
0.70	0.828	52	0.484
0.80	0.774	45	0.415
0.90	0.730	39	0.356
1.00	0.692	35	0.318
1.25	0.619	22	0.196
1.50	0.565	15	0.133
1.75	0.523	8	0.070
2.00	0.490	5	0.044
2.50	0.438	2	0.017
3.00	0.400	1	0.009
4.00	0.346	0	0.000

TABLE 7
PHYSICAL EXPERIMENT TEST III-5

$X = 5$ inches
 $\mu_0 = 1.12$ poise
 $\frac{R_0}{R_1} = 0.79$

Time Seconds	η	$P-P_0$ mm.Hg.	$\phi(\eta)$
0.00	∞	225	1.000
0.01	0.710	220	0.966
0.02	0.502	197	0.820
0.03	0.410	160	0.617
0.04	0.355	113	0.400
0.05	0.318	85	0.288
0.06	0.290	72	0.239
0.07	0.268	53	0.171
0.08	0.251	25	0.078
0.09	0.237	5	0.015
0.10	0.225	0	0.000

TABLE 8

PHYSICAL EXPERIMENT TEST III-15

$X = 15$ inches

$$\frac{R_0}{R_1} = 0.79$$

$\mu_0 = 1.12$ poise

Time Seconds	η	$P-P_0$ mm.Hg.	$\phi(\eta)$
0.00	∞	225	1.000
0.01	2.130	225	1.000
0.02	1.506	223	0.986
0.03	1.230	220	0.966
0.04	1.065	213	0.919
0.05	0.953	200	0.838
0.06	0.870	182	0.733
0.07	0.805	167	0.653
0.08	0.753	154	0.587
0.09	0.710	140	0.520
0.10	0.674	125	0.452
0.11	0.642	110	0.388
0.12	0.615	98	0.339
0.13	0.591	88	0.300
0.14	0.569	78	0.262
0.15	0.550	70	0.232
0.16	0.533	60	0.196
0.18	0.502	46	0.147
0.20	0.476	35	0.110
0.22	0.454	26	0.081
0.24	0.435	19	0.059
0.28	0.403	10	0.030
0.35	0.360	3	0.009
0.45	0.318	0	0.000

TABLE 9

PHYSICAL EXPERIMENT TEST III-25

$X = 25$ inches $\frac{R_0}{R_1} = 0.79$

$\mu_0 = 1.12$ poise

Time Seconds	η	$P-P_0$ mm.Hg.	$\phi(\eta)$
0.00	∞	225	1.000
0.10	1.123	225	1.000
0.11	1.071	221	0.973
0.12	1.025	215	0.932
0.13	0.985	210	0.900
0.14	0.949	203	0.856
0.15	0.917	197	0.820
0.16	0.888	191	0.784
0.17	0.861	184	0.744
0.18	0.837	177	0.706
0.19	0.815	170	0.668
0.20	0.794	165	0.642
0.22	0.757	154	0.587
0.24	0.725	144	0.539
0.26	0.696	135	0.497
0.28	0.671	126	0.456
0.30	0.648	117	0.417
0.32	0.628	110	0.388
0.34	0.609	101	0.351
0.36	0.592	94	0.323
0.40	0.561	80	0.269
0.45	0.529	65	0.214
0.50	0.502	52	0.168
0.55	0.479	41	0.130
0.60	0.458	33	0.104
0.70	0.424	20	0.062
0.80	0.397	11	0.034
1.00	0.355	2	0.006
1.20	0.324	0	0.000

TABLE 10
PHYSICAL EXPERIMENT TEST IV-5

$X = 5$ inches
 $\mu_0 = 1.12$ poise
 $\frac{R_0}{R_1} = 0.92$

Time Seconds	η	$P-P_0$ mm.Hg.	$\phi(\eta)$
0.00	∞	100	1.000
0.01	0.710	90	0.886
0.02	0.502	65	0.616
0.03	0.410	40	0.366
0.04	0.355	29	0.261
0.05	0.318	18	0.160
0.06	0.290	7	0.061
0.07	0.268	0	0.000

TABLE 11
PHYSICAL EXPERIMENT TEST IV-15

$X = 15$ inches
 $\mu_0 = 1.12$ poise
 $\frac{R_0}{R_1} = 0.92$

Time Seconds	η	P-P ₀ mm.Hg.	$\phi(\eta)$
0.00	∞	100	1.000
0.01	2.130	99	0.988
0.02	1.506	95	0.942
0.03	1.230	82	0.797
0.04	1.065	72	0.690
0.05	0.953	66	0.627
0.06	0.870	60	0.565
0.07	0.805	51	0.474
0.08	0.753	44	0.405
0.09	0.710	38	0.347
0.10	0.674	34	0.309
0.12	0.615	25	0.224
0.14	0.569	19	0.169
0.16	0.533	14	0.124
0.18	0.502	10	0.088
0.20	0.476	8	0.070
0.22	0.454	5	0.044
0.24	0.435	3	0.026
0.26	0.418	2	0.017
0.28	0.403	1	0.009
0.30	0.389	0	0.000

TABLE 12
PHYSICAL EXPERIMENT TEST IV-25

$X = 25$ inches
 $\mu_0 = 1.12$ poise
 $\frac{R_0}{R_1} = 0.92$

Time Seconds	η	$P-P_0$ mm.Hg.	$\phi(\eta)$
0.00	∞	100	1.000
0.03	2.050	100	1.000
0.04	1.775	99	0.988
0.05	1.588	97	0.965
0.06	1.450	94	0.931
0.07	1.342	90	0.886
0.08	1.255	87	0.852
0.09	1.184	85	0.830
0.10	1.123	80	0.775
0.12	1.025	71	0.679
0.14	0.949	62	0.585
0.16	0.888	56	0.524
0.18	0.837	50	0.464
0.20	0.794	44	0.405
0.22	0.757	39	0.356
0.24	0.725	35	0.318
0.26	0.696	31	0.280
0.30	0.648	24	0.215
0.35	0.600	17	0.151
0.40	0.561	12	0.106
0.45	0.529	9	0.079
0.50	0.502	5	0.044
0.55	0.479	4	0.035
0.60	0.458	1	0.009
0.70	0.424	0	0.000

TABLE 13

PHYSICAL EXPERIMENT TEST V-5

$X = 5$ inches
 $\mu_0 = 10.80$ poise
 $\frac{R_0}{R_1} = 0.79$

Time Seconds	η	$P-P_0$ mm.Hg.	$\phi(\eta)$
0.00	∞	225	1.000
0.02	1.563	225	1.000
0.03	1.276	224	0.993
0.04	1.105	222	0.979
0.05	0.988	220	0.966
0.06	0.902	217	0.946
0.07	0.835	214	0.926
0.08	0.781	210	0.900
0.09	0.737	207	0.875
0.10	0.699	202	0.850
0.11	0.666	198	0.826
0.12	0.638	195	0.808
0.13	0.613	192	0.790
0.14	0.591	187	0.761
0.16	0.553	180	0.722
0.18	0.521	172	0.679
0.20	0.494	165	0.642
0.22	0.471	158	0.607
0.24	0.451	152	0.577
0.26	0.433	147	0.553
0.30	0.404	135	0.497
0.35	0.374	124	0.448
0.40	0.349	113	0.400
0.50	0.313	95	0.327
0.75	0.255	60	0.196
1.00	0.221	36	0.114
1.50	0.180	10	0.030
3.00	0.128	0	0.000

TABLE 14
PHYSICAL EXPERIMENT TEST V-15

X = 15 inches
 $\mu_0 = 10.80$ poise
 $\frac{R_0}{R_1} = 0.79$

Time Seconds	η	P-P ₀ mm.Hg.	$\phi(\eta)$
0.00	∞	225	1.000
0.07	2.506	225	1.000
0.10	2.097	224	0.993
0.13	1.839	223	0.986
0.15	1.712	222	0.979
0.17	1.608	221	0.973
0.19	1.521	220	0.966
0.21	1.447	219	0.959
0.23	1.383	217	0.946
0.25	1.326	215	0.932
0.27	1.276	213	0.919
0.30	1.211	210	0.900
0.33	1.154	207	0.881
0.36	1.105	204	0.862
0.40	1.048	200	0.838
0.45	0.988	194	0.802
0.50	0.938	188	0.767
0.60	0.856	176	0.700
0.70	0.793	166	0.647
0.80	0.741	156	0.597
1.00	0.663	138	0.511
1.25	0.593	117	0.417
1.50	0.541	99	0.343
2.00	0.469	68	0.225
3.00	0.383	30	0.094
4.00	0.332	10	0.030
5.00	0.297	2	0.006
7.00	0.251	0	0.000

TABLE 15
PHYSICAL EXPERIMENT TEST V-25

$X = 25$ inches
 $\mu_0 = 10.80$ poise
 $\frac{R_0}{R_1} = 0.79$

Time Seconds	η	$P-P_0$ mm.Hg.	$\phi(\eta)$
0.00	∞	225	1.000
0.08	3.907	225	1.000
0.20	2.471	224	0.993
0.40	1.747	223	0.986
0.46	1.629	222	0.979
0.50	1.563	221	0.973
0.60	1.427	220	0.966
0.70	1.321	217	0.946
0.80	1.236	215	0.932
0.90	1.165	211	0.906
1.00	1.105	208	0.887
1.20	1.009	200	0.838
1.40	0.934	193	0.796
1.60	0.874	186	0.756
1.80	0.824	178	0.700
2.00	0.781	170	0.668
2.50	0.699	125	0.577
3.00	0.638	135	0.497
3.50	0.591	118	0.422
4.00	0.553	103	0.359
5.00	0.494	75	0.250
6.00	0.451	54	0.175
7.00	0.418	37	0.117
8.00	0.391	25	0.078

TABLE 16
PHYSICAL EXPERIMENT TEST VI-5

$X = 5$ inches
 $\mu_0 = 10.80$ poise
 $\frac{R_0}{R_1} = 0.92$

Time Seconds	η	$P-P_0$ mm.Hg.	$\phi(\eta)$
0.00	∞	100	1.000
0.03	1.126	100	1.000
0.04	1.105	99	0.988
0.05	0.988	97	0.965
0.06	0.902	96	0.954
0.07	0.835	94	0.931
0.08	0.781	92	0.908
0.09	0.737	90	0.886
0.10	0.699	89	0.875
0.12	0.638	85	0.830
0.14	0.591	82	0.797
0.16	0.553	78	0.754
0.18	0.521	74	0.711
0.20	0.494	70	0.668
0.25	0.442	60	0.565
0.30	0.404	51	0.474
0.35	0.374	44	0.405
0.40	0.349	37	0.337
0.45	0.329	31	0.280
0.50	0.313	25	0.224
0.60	0.285	17	0.151
0.70	0.264	10	0.088
0.80	0.247	5	0.044
0.90	0.233	2	0.017
1.10	0.211	0	0.000

TABLE 17

PHYSICAL EXPERIMENT TEST VI-15

$X = 15$ inches
 $\mu_0 = 10.80$ poise
 $\frac{R_0}{R_1} = 0.92$

Time Seconds	η	$P-P_0$ mm.Hg.	$\phi(\eta)$
0.00	∞	100	1.000
0.15	1.712	100	1.000
0.17	1.608	99	0.988
0.19	1.521	98	0.977
0.22	1.414	96	0.954
0.25	1.326	95	0.942
0.28	1.253	94	0.931
0.31	1.191	92	0.908
0.34	1.137	90	0.886
0.38	1.076	88	0.864
0.43	1.011	85	0.830
0.47	0.967	82	0.797
0.50	0.938	80	0.775
0.55	0.894	77	0.743
0.60	0.856	74	0.711
0.70	0.793	67	0.637
0.80	0.741	61	0.575
0.91	0.695	55	0.514
1.01	0.660	50	0.464
1.25	0.593	39	0.356
1.50	0.541	30	0.271
1.75	0.501	22	0.196
2.00	0.469	16	0.142
2.50	0.419	8	0.070
3.00	0.383	3	0.026
3.50	0.354	1	0.009
4.50	0.313	0	0.000

TABLE 18

PHYSICAL EXPERIMENT TEST VI-25

X = 25 inches

$\mu_0 = 10.80$ poise

$\frac{R_0}{R_1} = 0.92$

Time Seconds	η	P-P ₀ mm.Hg.	$\phi(\eta)$
0.00	∞	100	1.000
0.30	2.018	100	1.000
0.34	1.895	99	0.988
0.40	1.747	97	0.965
0.46	1.629	96	0.954
0.50	1.563	95	0.942
0.56	1.477	93	0.920
0.60	1.427	92	0.908
0.66	1.360	90	0.886
0.80	1.236	85	0.830
0.94	1.140	80	0.775
1.20	1.009	70	0.668
1.48	0.908	60	0.565
1.82	0.819	50	0.464
2.00	0.781	45	0.415
2.40	0.713	35	0.318
2.94	0.645	25	0.224
3.68	0.576	15	0.133
4.14	0.543	10	0.088
4.82	0.503	5	0.044
6.00	0.450	0	0.000

TABLE 19
PHYSICAL EXPERIMENT TEST VII-25

X = 25 inches
 $\mu = 10.80$ poise
 $\frac{R_0}{R_1} = 0.79$

NOTE: Test performed without lubricant

Time Seconds	η	P-P ₀ mm.Hg.	$\phi(\eta)$
0.00	∞	225	1.000
0.22	2.356	225	1.000
0.30	2.018	223	0.986
0.40	1.747	220	0.966
0.46	1.629	217	0.946
0.50	1.563	215	0.932
0.60	1.427	210	0.900
0.80	1.236	199	0.832
1.00	1.105	187	0.761
1.50	0.902	157	0.602
2.00	0.781	132	0.483
2.50	0.699	107	0.375
3.00	0.638	87	0.296
3.50	0.591	70	0.232
4.00	0.553	55	0.178
5.00	0.494	34	0.107
6.00	0.451	18	0.055
7.00	0.418	10	0.030
8.00	0.391	3	0.009

TABLE 20

PHYSICAL EXPERIMENT TEST VIII-15

X = 15 inches

$\mu = 6.68$ poise

$\frac{R_0}{R_1} = 0.79$

NOTE: Catheter to clamp length
extended to 18.5 inches to produce
"semi-infinite" tube.

Time Seconds	η	$P-P_0$ mm.Hg.	$\phi(\eta)$
0.00	∞	223	1.000
0.06	2.124	223	1.000
0.10	1.645	221	0.986
0.13	1.443	220	0.979
0.16	1.301	216	0.952
0.18	1.226	214	0.939
0.20	1.163	212	0.926
0.24	1.062	208	0.900
0.28	0.983	201	0.856
0.32	0.920	196	0.825
0.36	0.867	192	0.801
0.41	0.813	186	0.766
0.47	0.759	180	0.732
0.61	0.666	169	0.672
0.74	0.605	158	0.615
0.97	0.528	143	0.541
1.24	0.467	123	0.476
1.54	0.419	115	0.415
1.97	0.371	94	0.328
2.48	0.330	74	0.250
3.05	0.298	55	0.181
3.60	0.274	40	0.129
4.65	0.241	20	0.063
7.80	0.000	0	0.000

TABLE 21

PHYSICAL EXPERIMENT TEST IX-15

X = 15 inches

$\mu = 6.68$ poise

$\frac{R_0}{R_1}$

= 0.79

NOTE: Catheter to clamp length
extended to 18.5 inches to produce
"semi-infinite" tube.

Time Seconds	η	P-P ₀ mm.Hg.	$\phi(\eta)$
0.00	∞	225	1.000
0.10	1.645	220	0.966
0.15	1.343	218	0.952
0.19	1.194	215	0.932
0.23	1.085	210	0.900
0.27	1.001	205	0.868
0.31	0.934	200	0.838
0.35	0.879	195	0.808
0.44	0.784	185	0.750
0.49	0.743	180	0.722
0.60	0.672	170	0.668
0.70	0.622	162	0.627
0.80	0.582	156	0.597
0.90	0.548	150	0.567
1.25	0.465	137	0.506
1.50	0.425	120	0.430
2.00	0.368	99	0.343
2.25	0.347	89	0.303
2.50	0.329	80	0.269
3.00	0.300	63	0.207
4.00	0.260	35	0.110
5.00	0.233	18	0.055
7.00	0.197	2	0.006

TABLE 21

PHYSICAL EXPERIMENT TEST IX-15

X = 15 inches

$\mu = 6.68 \text{ poise}$

$\frac{R_0}{R_1} = 0.79$

NOTE: Catheter to clamp length
extended to 18.5 inches to produce
"semi-infinite" tube.

Time Seconds	η	P-P ₀ mm.Hg.	$\phi(\eta)$
0.00	∞	225	1.000
0.10	1.645	220	0.966
0.15	1.343	218	0.952
0.19	1.194	215	0.932
0.23	1.085	210	0.900
0.27	1.001	205	0.868
0.31	0.934	200	0.838
0.35	0.879	195	0.808
0.44	0.784	185	0.750
0.49	0.743	180	0.722
0.60	0.672	170	0.668
0.70	0.622	162	0.627
0.80	0.582	156	0.597
0.90	0.548	150	0.567
1.25	0.465	137	0.506
1.50	0.425	120	0.430
2.00	0.368	99	0.343
2.25	0.347	89	0.303
2.50	0.329	80	0.269
3.00	0.300	63	0.207
4.00	0.260	35	0.110
5.00	0.233	18	0.055
7.00	0.197	2	0.006

TABLE 23
PHYSIOLOGICAL EXPERIMENT LEFT FEMORAL VEIN
CLAMPED DISTAL TO POINT OF SEVERANCE

$X = 1.0 \text{ inches}$
 $R_1 = 0.049 \text{ inches}$

$\frac{R_0}{R_1} = 0.3 \dots \text{Assumed}$

Time Seconds	η	P-P ₀ mm.Hg.	$\phi(\eta)$
0.00	∞	4.85	1.000
0.06	0.246	4.85	1.000
0.07	0.228	4.82	0.998
0.08	0.213	4.80	0.997
0.10	0.191	4.75	0.993
0.12	0.174	4.68	0.988
0.16	0.151	4.50	0.974
0.20	0.135	4.28	0.956
0.24	0.123	3.88	0.919
0.30	0.110	3.38	0.863
0.50	0.085	2.36	0.710
2.00	0.043	1.20	0.442

TABLE 24

PHYSIOLOGICAL EXPERIMENT LEFT SAPHENOUS VEIN
NO CLAMP DISTAL TO POINT OF SEVERANCE

$X = 1.8 \text{ inches}$
 $R_1 = 0.059 \text{ inches}$

$\frac{R_0}{R_1} = 0.3 \dots \text{Assumed}$

Time Seconds	η	$P-P_0$ mm.Hg.	$\phi(\eta)$
0.0	∞	9.50	1.000
0.55	0.156	9.50	1.000
0.60	0.149	9.45	1.000
0.65	0.144	9.25	0.997
0.70	0.138	9.05	0.995
0.75	0.134	8.90	0.993
0.80	0.129	8.70	0.991
1.00	0.116	8.00	0.980
1.50	0.095	7.35	0.967
2.00	0.082	6.85	0.955
2.50	0.073	6.55	0.946
4.00	0.058	6.00	0.928

TABLE 25
PHYSIOLOGICAL EXPERIMENT INTACT LEFT FEMORAL VEIN
NO CLAMP DISTAL TO POINT OF SEVERANCE

$X = 1.5 \text{ inches}$
 $R_1 = 0.059 \text{ inches}$

$\frac{R_0}{R_1} = 0.3 \dots \text{Assumed}$

Time Seconds	η	$P-P_0$ mm.Hg.	$\phi(\eta)$
0.00	∞	2.30	1.000
0.01	0.523	2.30	1.000
0.02	0.370	2.25	0.986
0.03	0.302	2.20	0.973
0.04	0.262	2.15	0.959
0.06	0.214	2.10	0.944
0.08	0.185	2.02	0.921
0.15	0.135	2.00	0.915
0.25	0.105	1.88	0.878
0.35	0.088	1.70	0.819
0.45	0.078	1.60	0.784
1.00	0.052	1.40	0.711

APPENDIX G
PRESSURE RECORDINGS

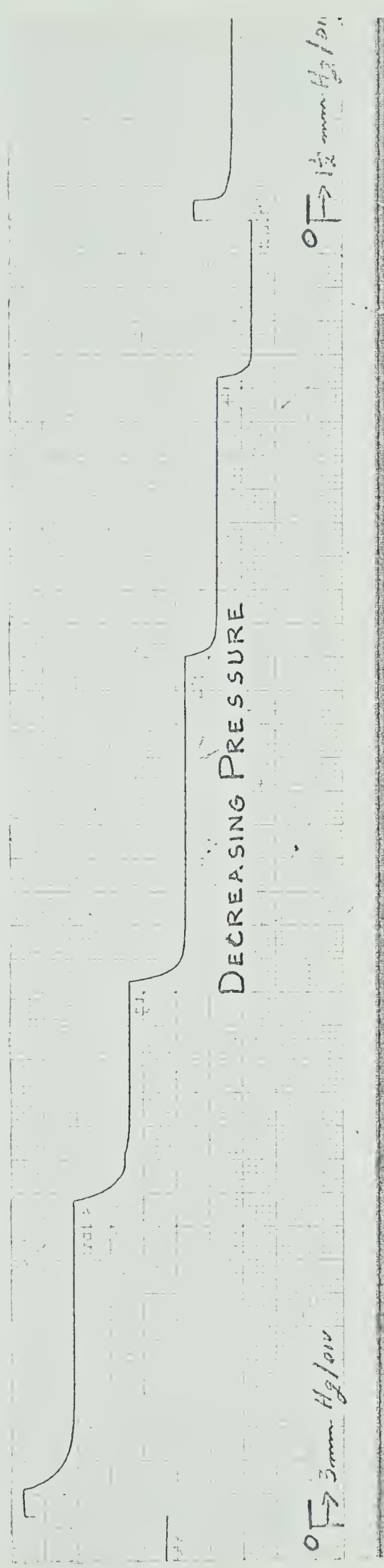
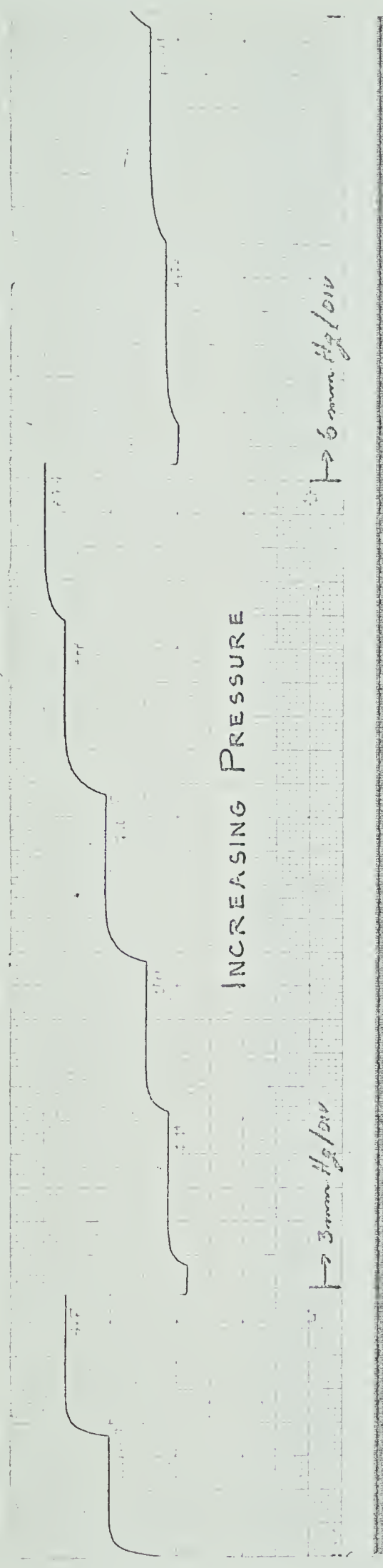


FIG. G.1 PRESSURE RECORD - STATIC TEST OF BLUE LATEX TUBE

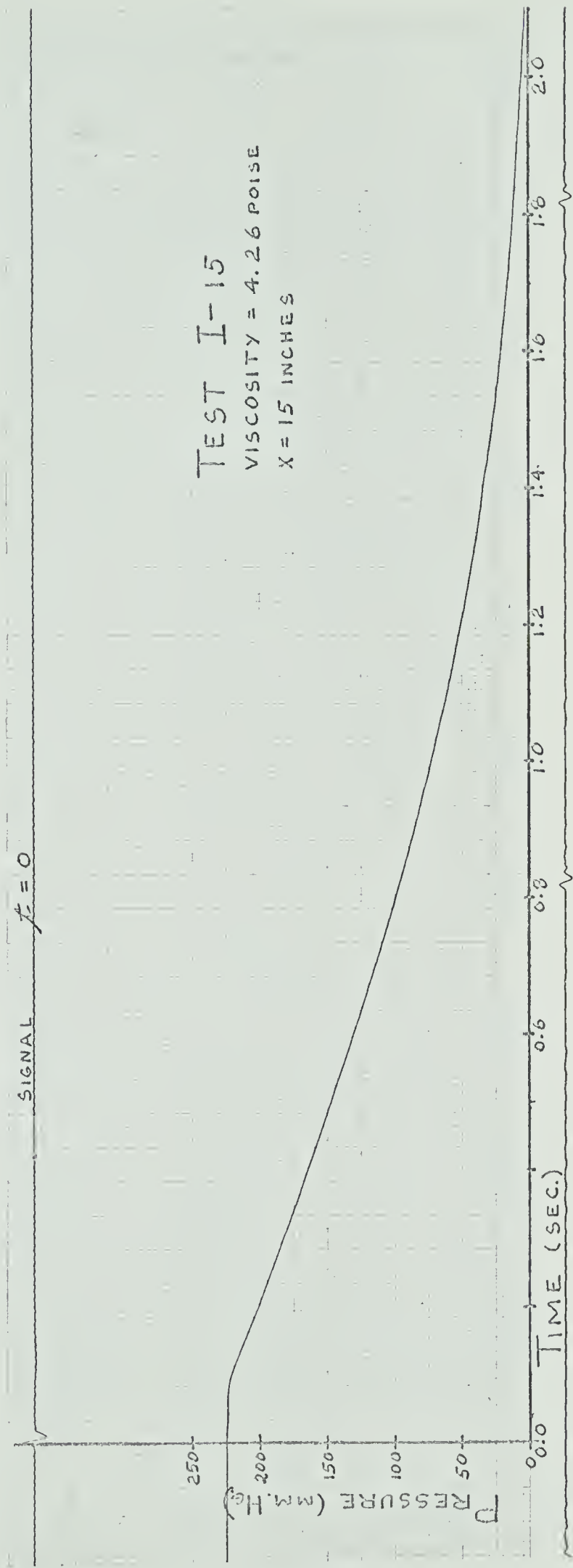


FIG. G.2 PRESSURE-TIME RECORD; TEST I-15

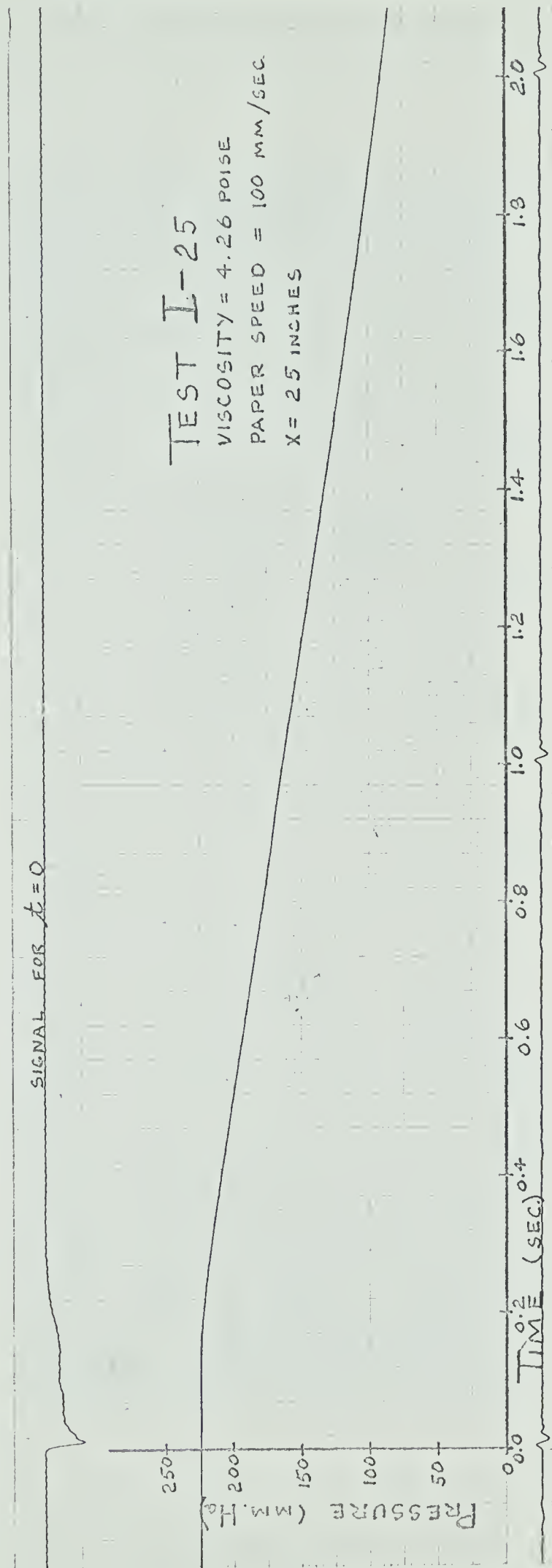


FIG. G.3 PRESSURE-TIME RECORD; TEST I-25

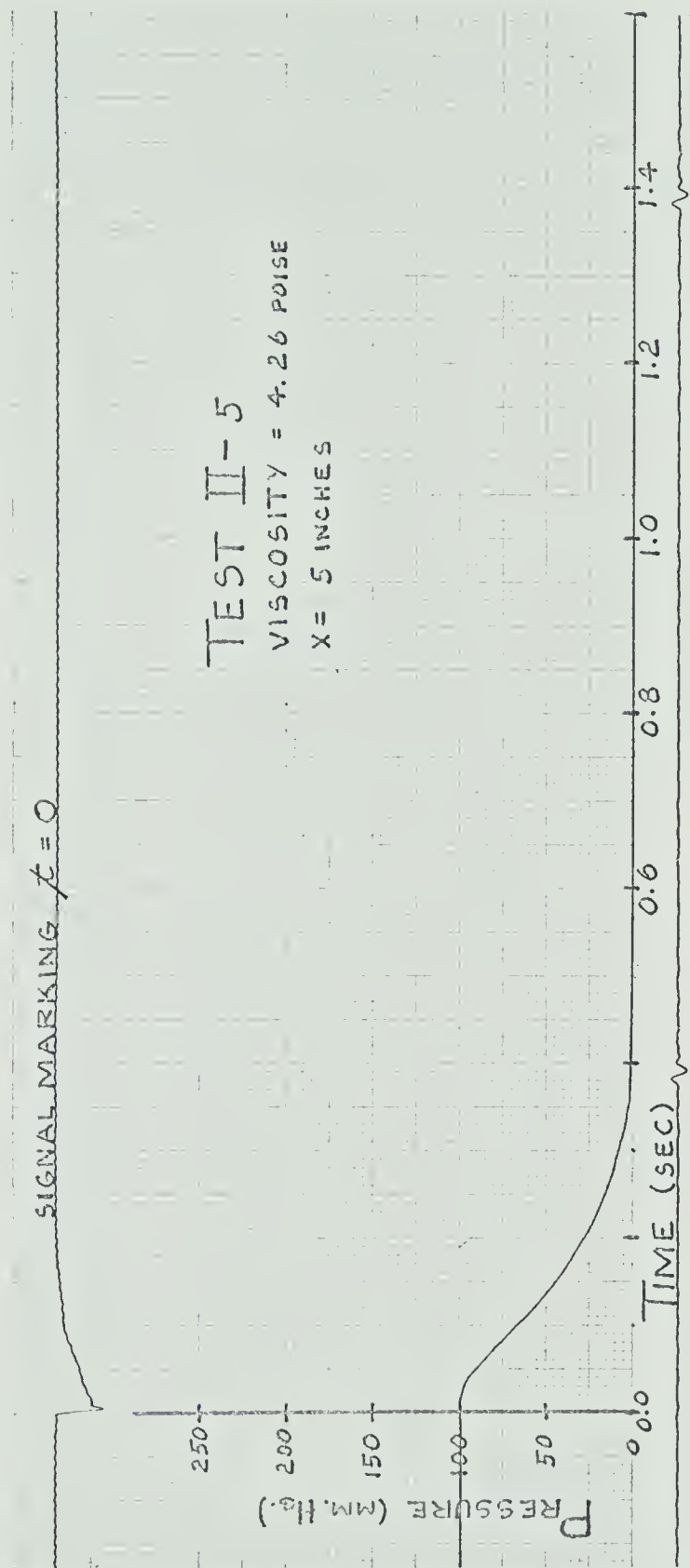


FIG. G.4 PRESSURE-TIME RECORD: TEST II-5

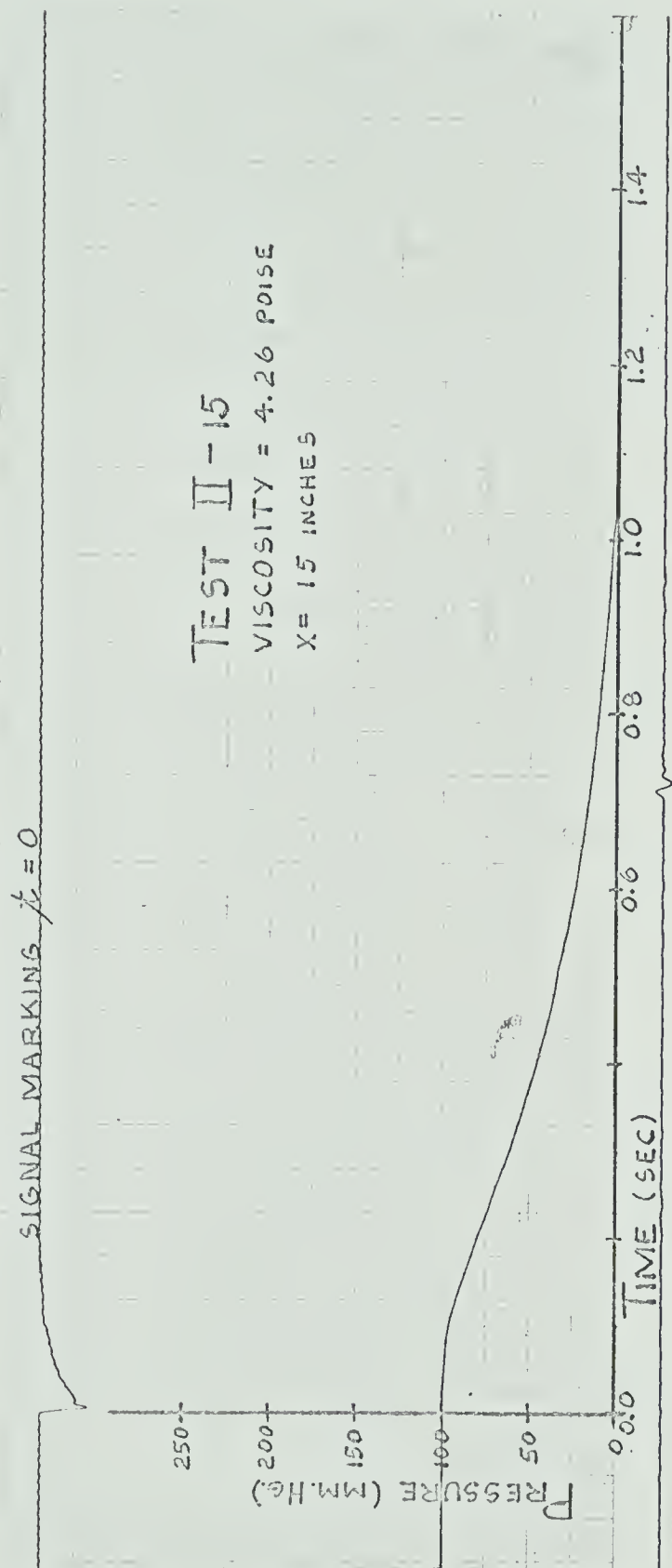


FIG. G.5 PRESSURE-TIME RECORD; TEST II-15

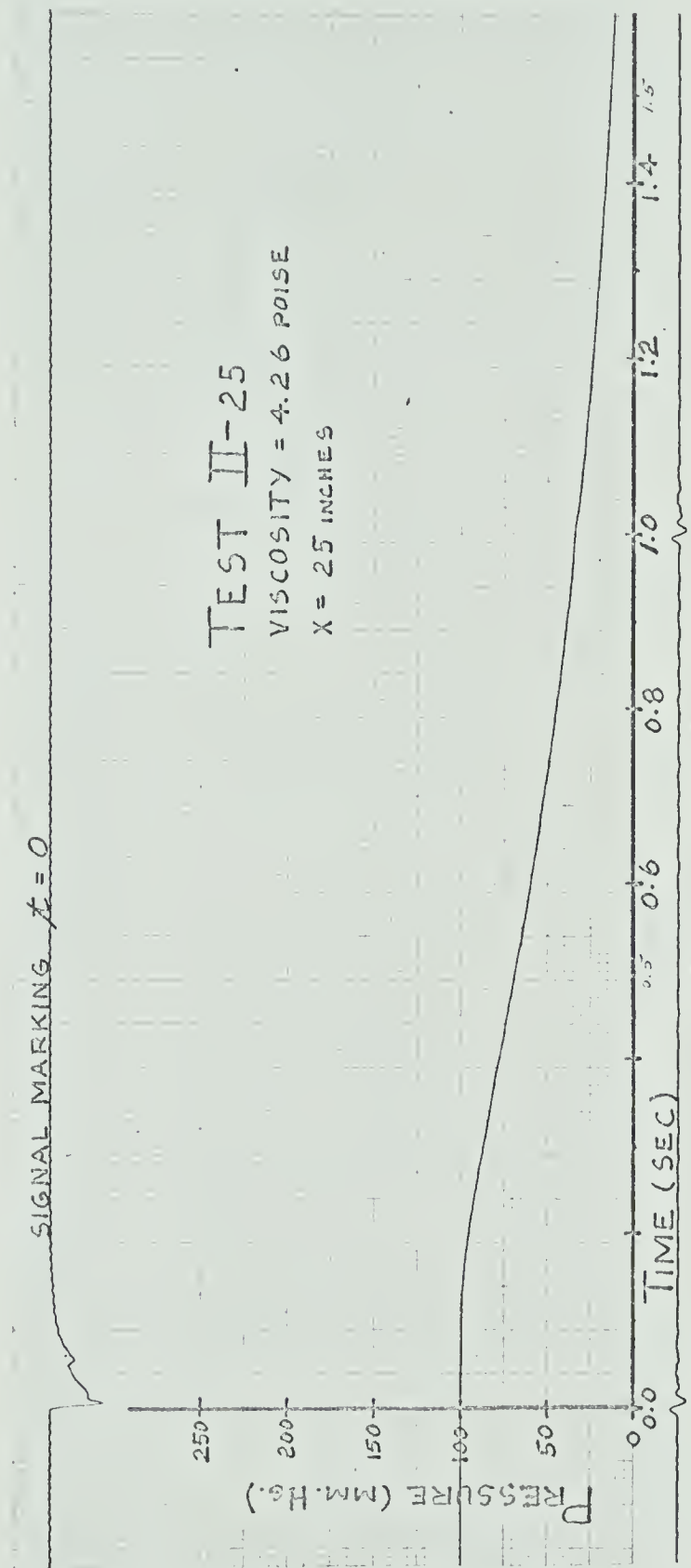


FIG. G.6 PRESSURE-TIME RECORD; TEST II-25

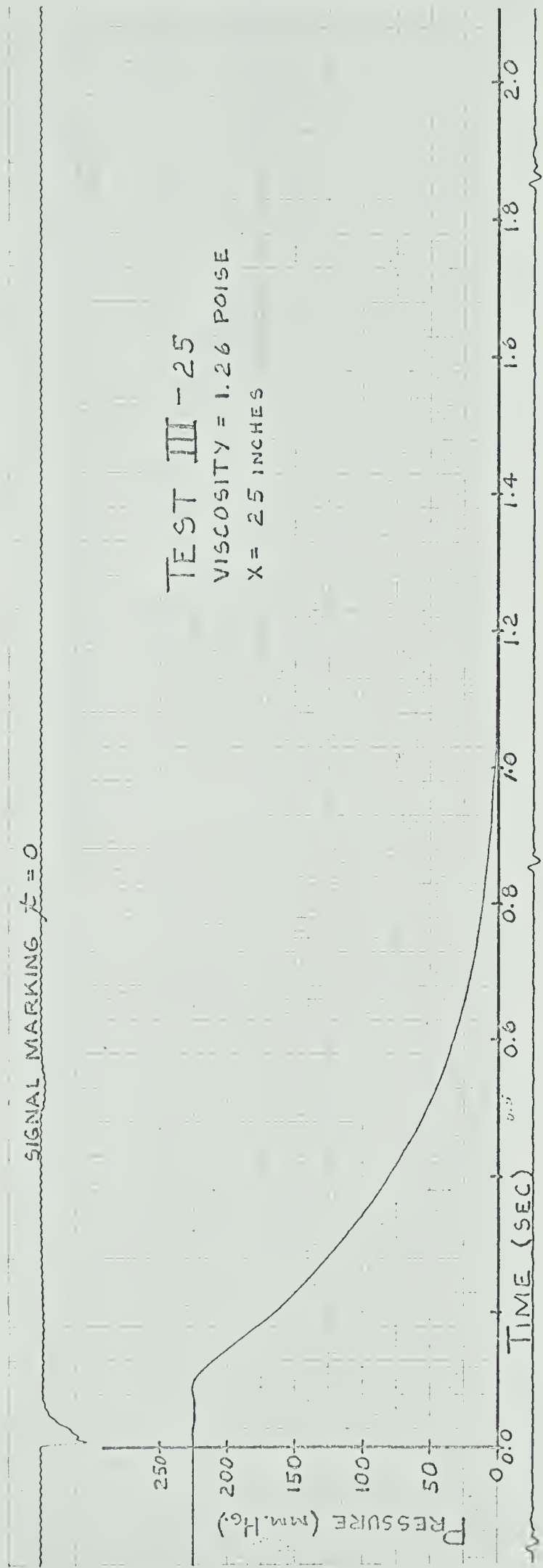


FIG. G.7 PRESSURE-TIME RECORD; TEST III-25

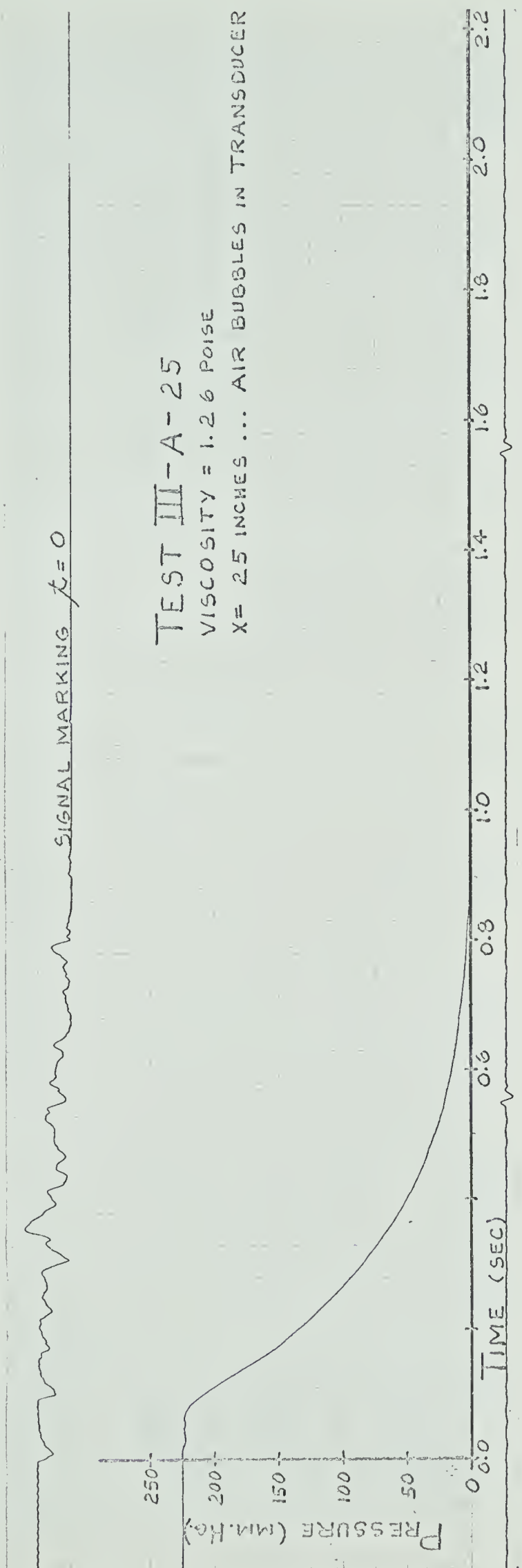


FIG. G.8 PRESSURE-TIME RECORD; TEST III-A-25, EFFECT OF AIR BUBBLES

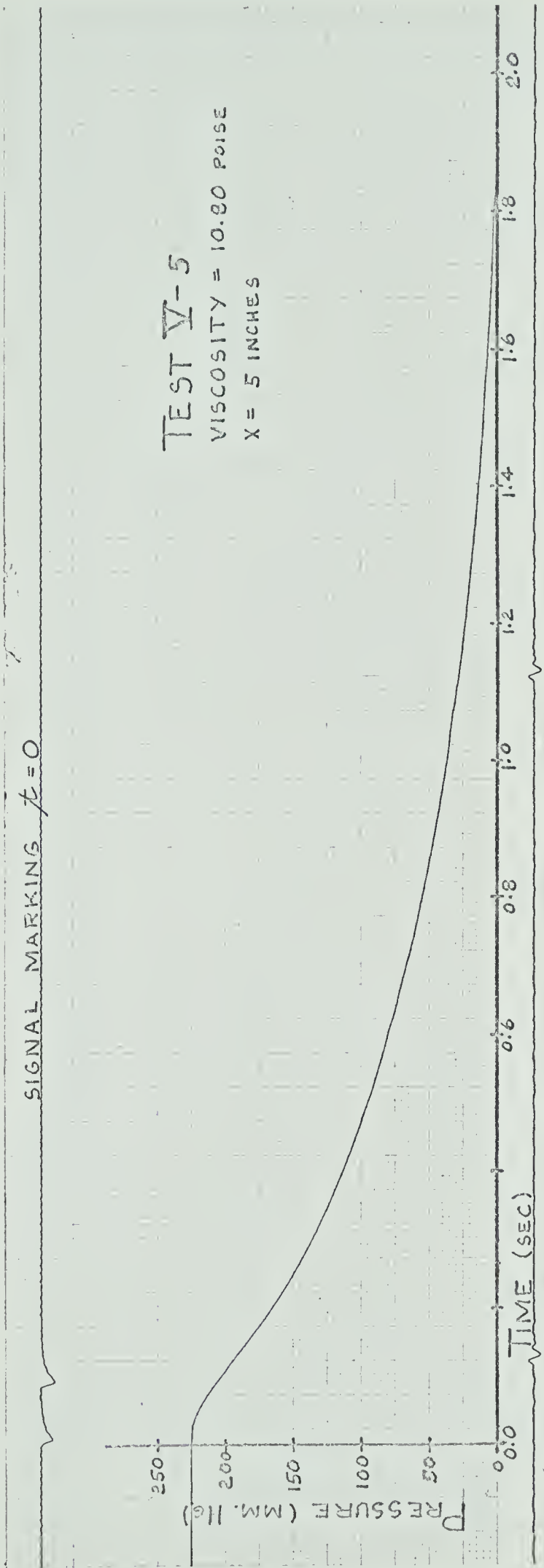


FIG. G.9 PRESSURE-TIME RECORD; TEST V-5

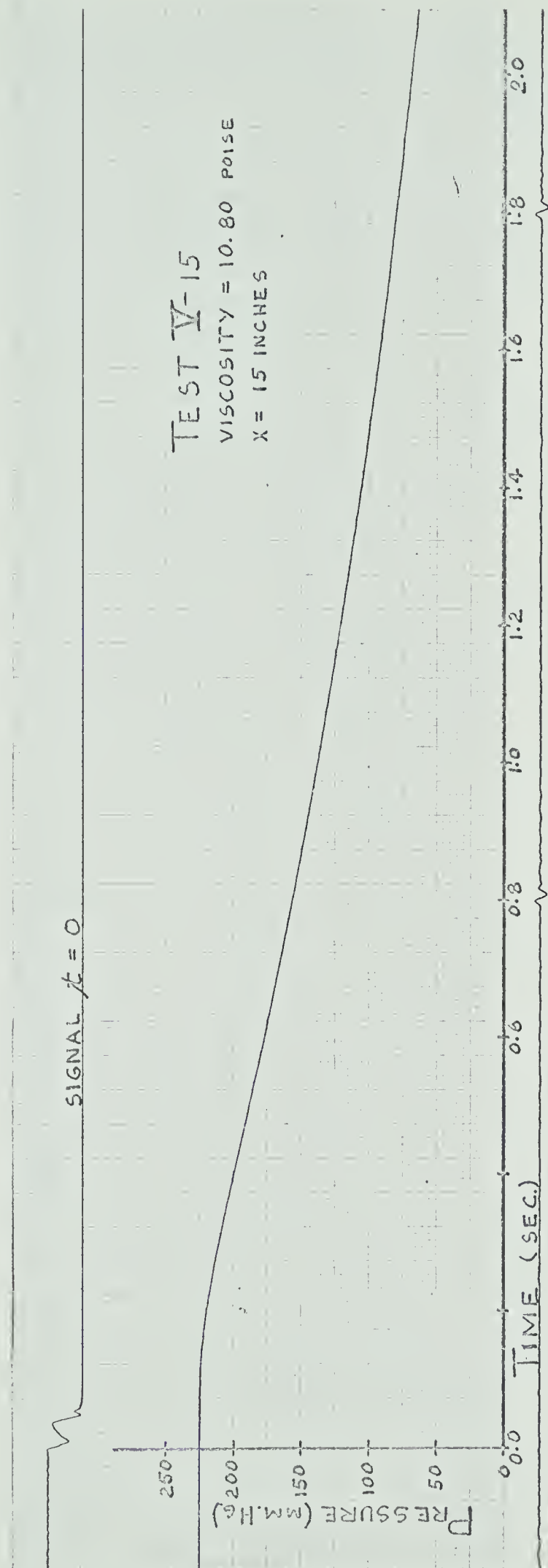


FIG. G.10 PRESSURE-TIME RECORD: TEST V-15

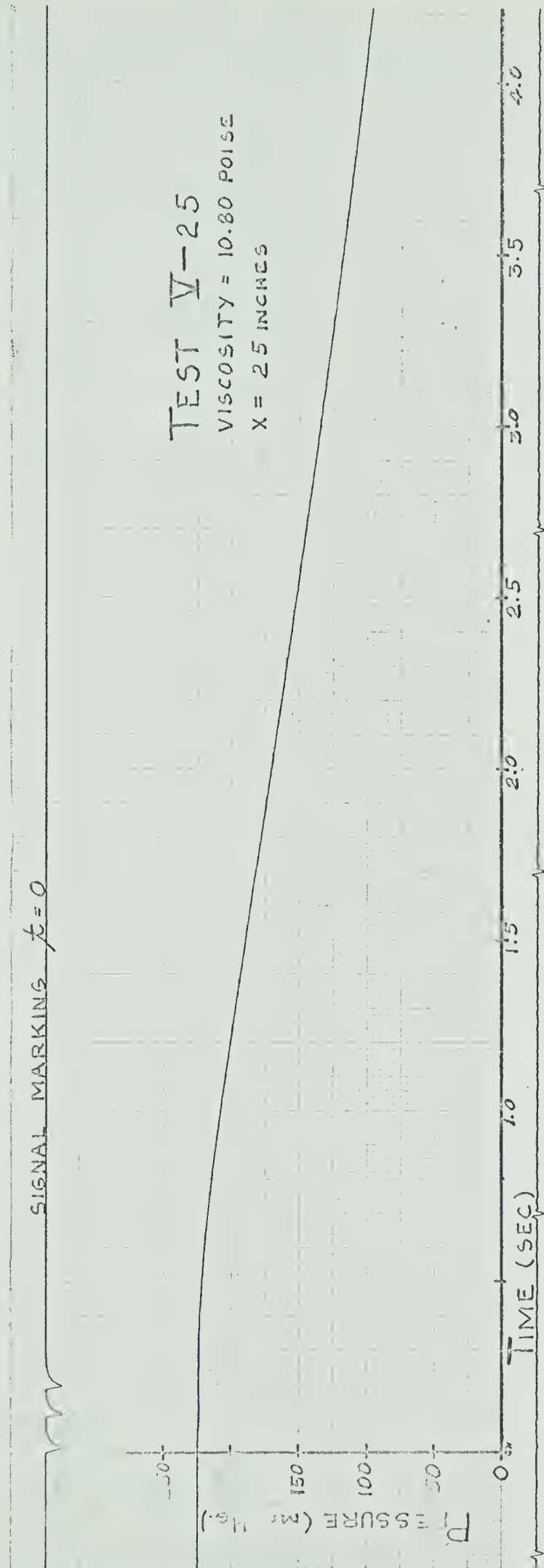


FIG. G.11 PRESSURE-TIME RECORD: TEST V-25

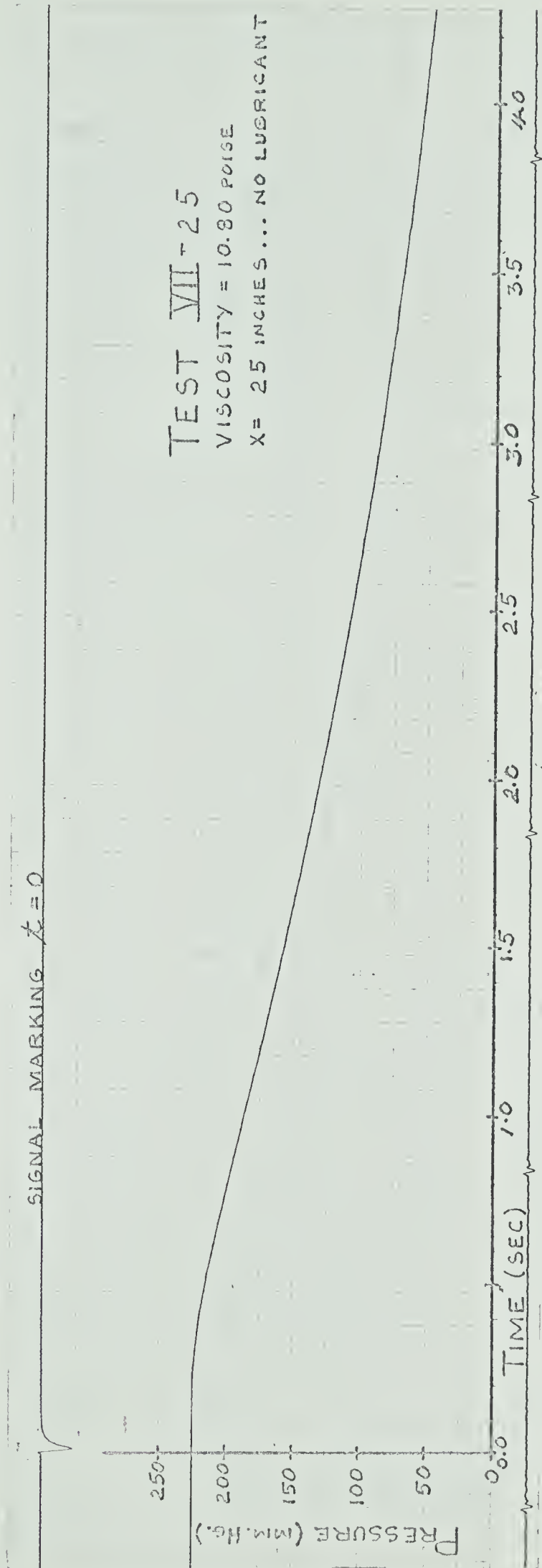


FIG. G.12 PRESSURE-TIME RECORD; TEST VII-25, EFFECT OF NO LUBRICANT

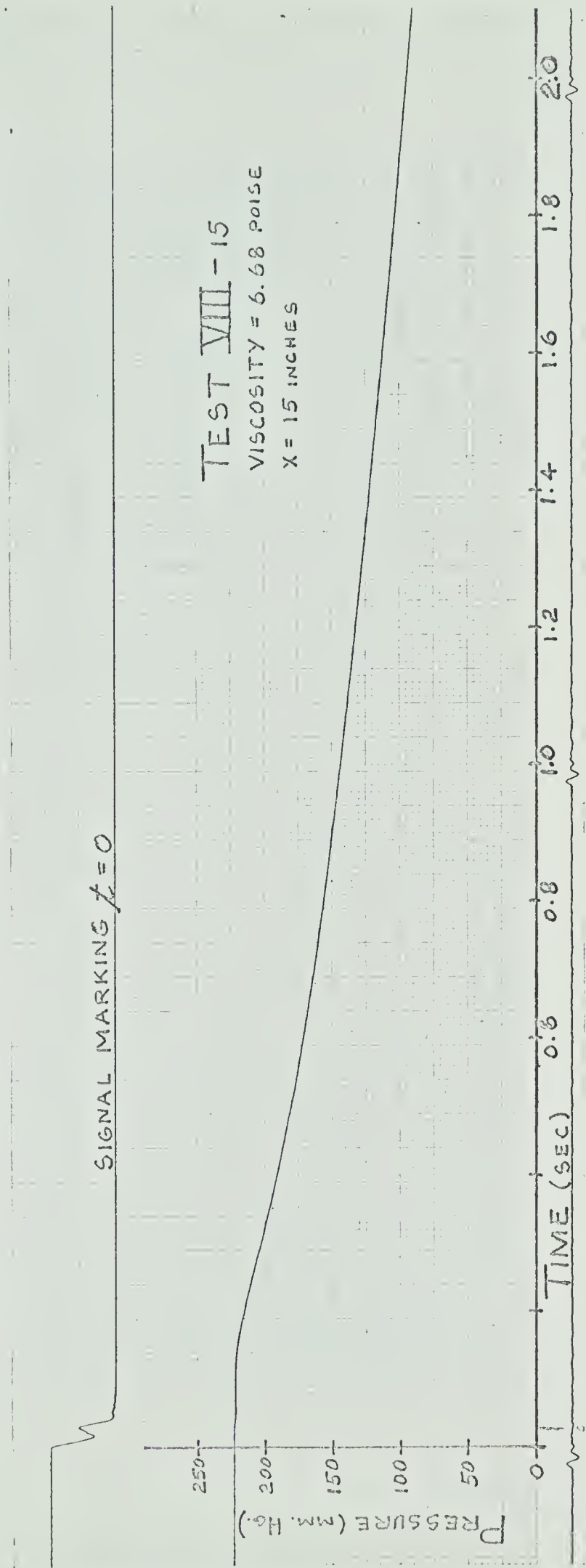


FIG. G.13 PRESSURE-TIME RECORD; TEST VIII-15, 'SEMI-INFINITE' TUBE

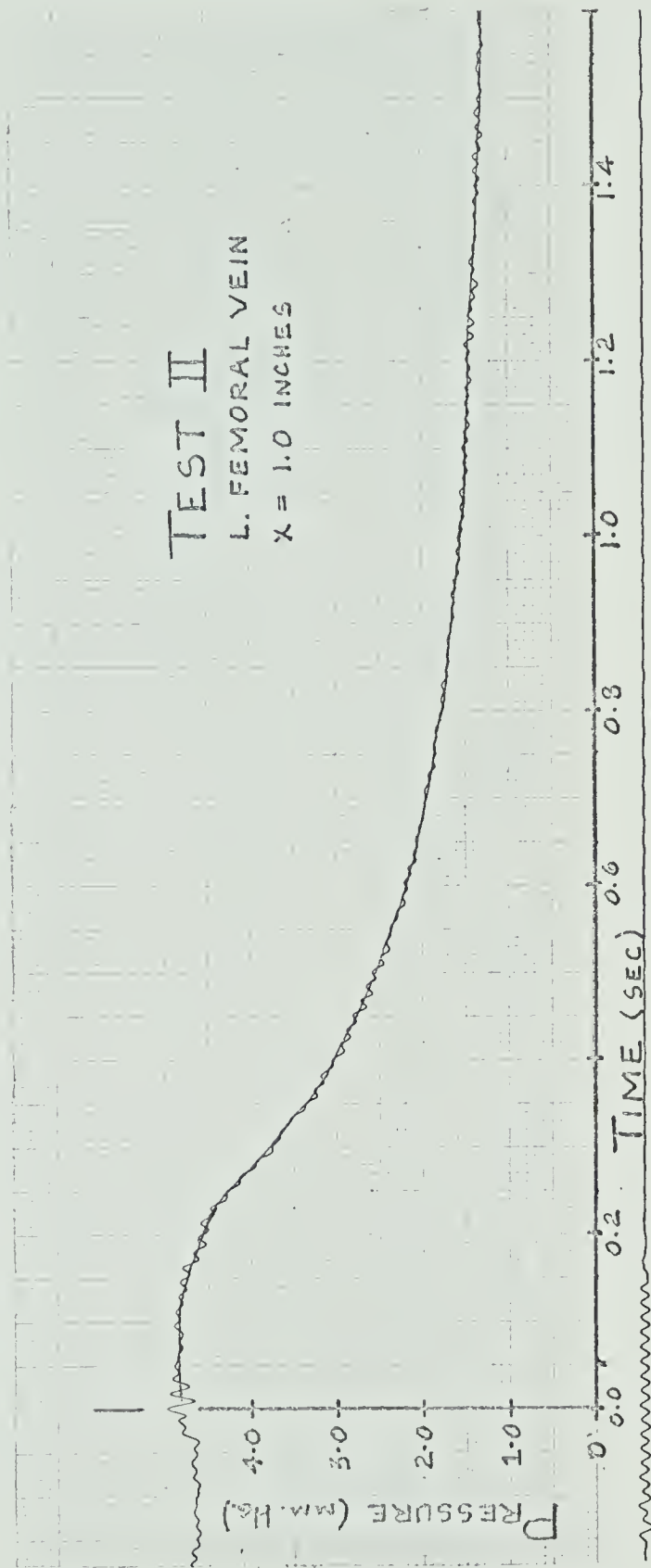


FIG. G.14 PRESSURE-TIME RECORD; LEFT FEMORAL VEIN

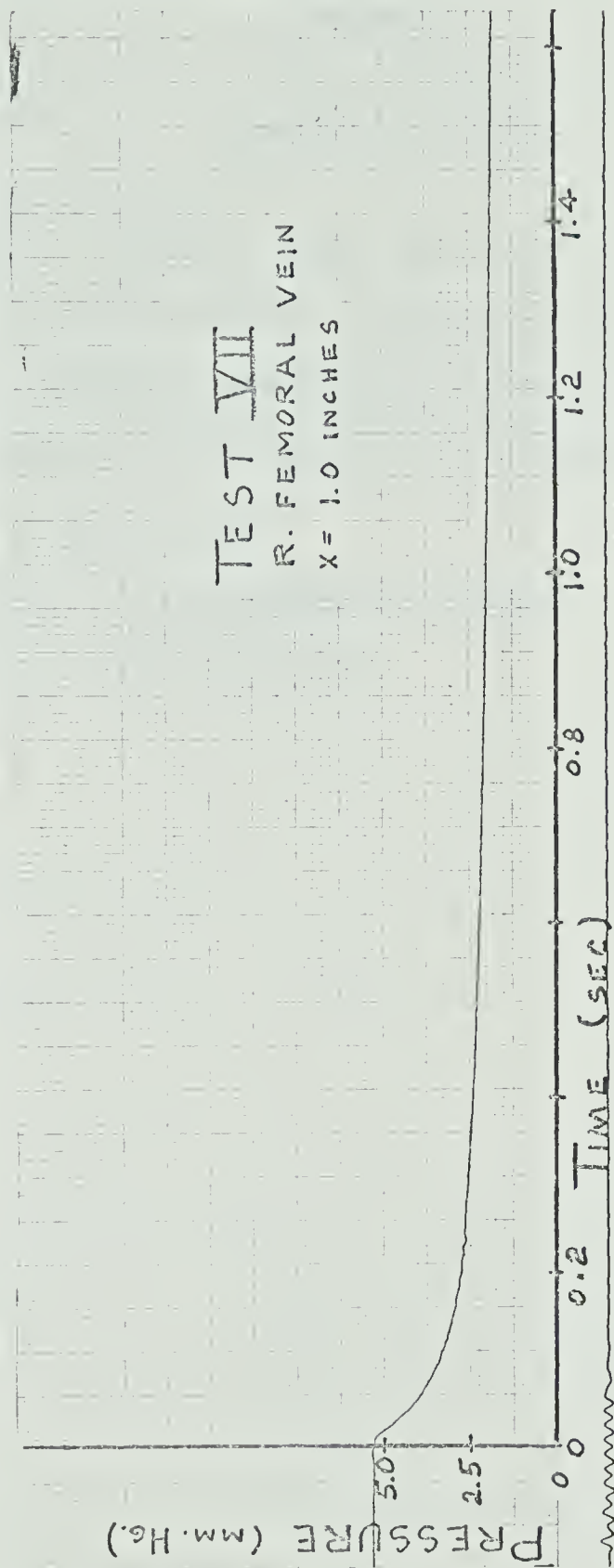


FIG. G.15 PRESSURE-TIME RECORD; RIGHT FEMORAL VEIN

APPENDIX H

CONVERSION FACTORS

Wherever possible, units reported in this thesis are consistent with the metric system. Departures are made from this system for material presented in the Appendices due to instrument scale readings and convenience during the test program. The following conversion factors can be used:

1 mm.Hg. = 13.6 kilograms/sq.meter

1 inch = 2.54 centimeters .

APPENDIX I

ERROR ANALYSIS

I.1 Physical Experiment

Pressure

- Calibration of pressure transducer including balancing error and non-linearity of recording system	<u>± 3.0 mm.Hg.</u>
- positioning transducer	<u>± 0.50 mm.Hg.</u>
- reading pressure trace	<u>± 0.50 mm.Hg.</u>
	<u>± 4.00 mm.Hg.</u>

Radius

- measuring ΔR	<u>± 0.005 inches</u>
- measuring wall thickness	<u>± 0.00025 inches</u>
	<u>± 0.00525 inches</u>

Viscosity

- reading dial gauge ± 0.5° deflection

This corresponds to	<u>± 0.015 poise at 100 RPM</u>
	<u>± 0.0075 poise at 200 RPM</u>
	<u>± 0.005 poise at 300 RPM</u>
	<u>± 0.0025 poise at 600 RPM</u>

Length (X)

- marking tubes and positioning guillotine ± 0.05 inches

Time (t)

- determination of $t = 0$ and delay in recording system ± 0.01 sec.

Hence the measured values of η should be within

$$\frac{2 \times 24.95}{0.1164 \times 0.00525} \sqrt{\frac{10.785 \times 0.002088}{3.1 \times 1055 \times t}} < \eta <$$

$$\frac{2 \times 25.05}{0.1164 - 0.00525} \sqrt{\frac{10.815 \times 0.002088}{3.1 \times 1055 \times t}}$$

If small values of time are considered, that is, large values of η , $\frac{1.0767}{\sqrt{0.01}} < \eta < \frac{1.184}{\sqrt{0.03}}$, $\eta \pm 1.96$. Thus accurate determination of time is essential in this region to obtain close agreement with theory. Very good agreement was noted between experiment and theory for large values of η .

Considering large values of time, $\frac{1.0767}{\sqrt{1.01}} < \eta < \frac{1.184}{\sqrt{1.03}}$, or $\eta \pm 0.048$. The range of experimental error is considerably narrowed. The deviation of experiment from theory was greatest for small η , the reasons for which are discussed earlier in the text. Successive trials under the same conditions found the experimental results to be within these limits of experimental error.

Using the expression from Appendix A.4,

$$(R-R_0)_{\max} = \frac{0.11715}{3.1} \ln \left(\frac{397}{397 - (225+8.0)} \right)$$

$$= .03323$$

$$(R-R_0)_{\max} = \frac{0.11715}{3.1} \ln \left(\frac{397}{397 - 225} \right)$$

$$= 0.03163$$

Thus measured values of ϕ should be within

$$\frac{.03323 - 0.03163}{.03323} = .048 \text{ or } \phi \pm .024.$$

I.2 Physiological Experiments

Pressure

- same as for I.1 ± 4.00 mm.Hg.

Radius - as shown by Figures I.1 and I.2, the assumption of $R_0/R_1 = 0.2$ and 0.4 rather than the 0.3 chosen does not affect the relative position of the experimental or theoretical curves.

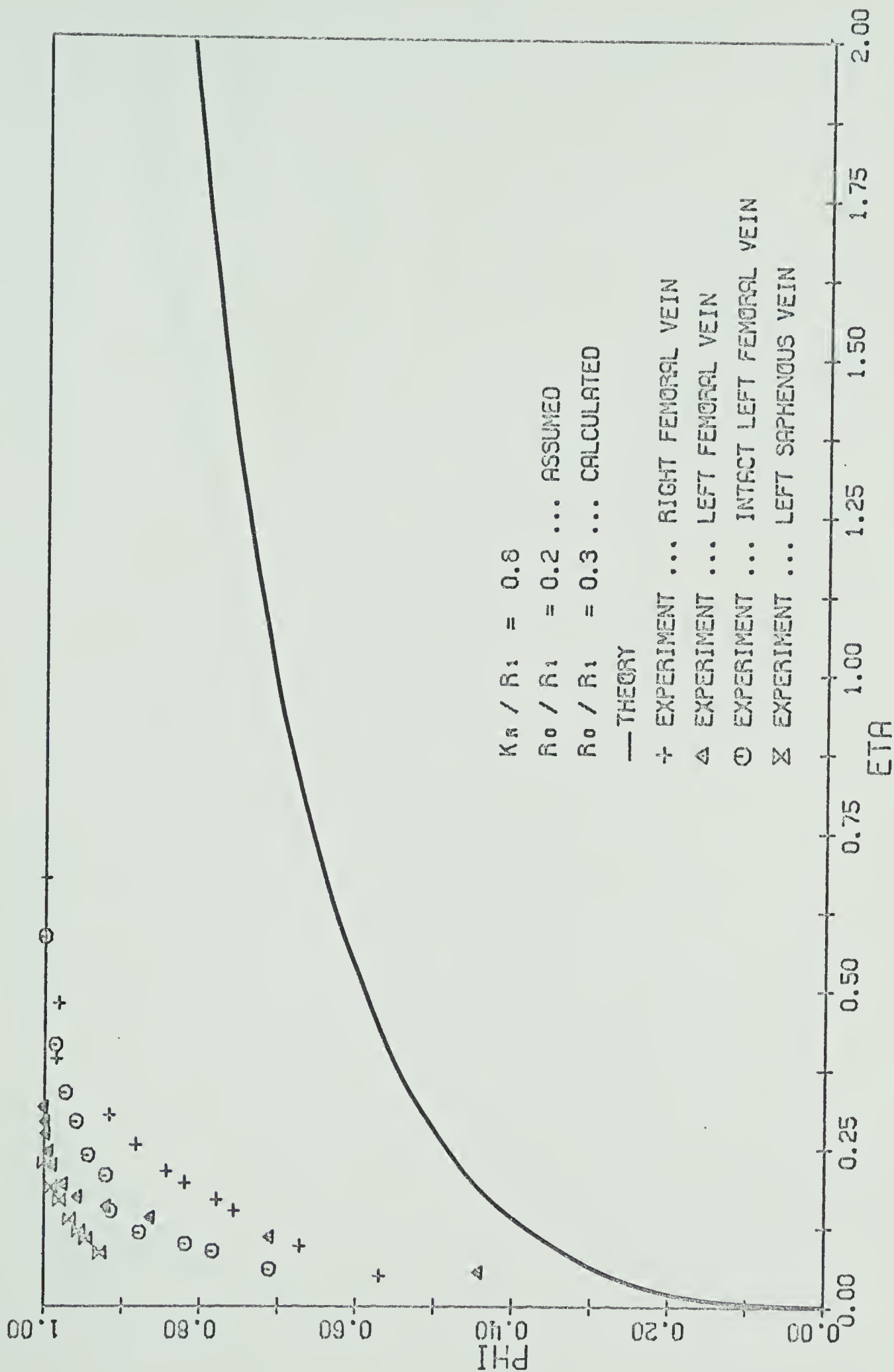


FIG. I.1 PHYSIOLOGICAL EXPERIMENTS ASSUMING $R_o/R_1 = 0.2$

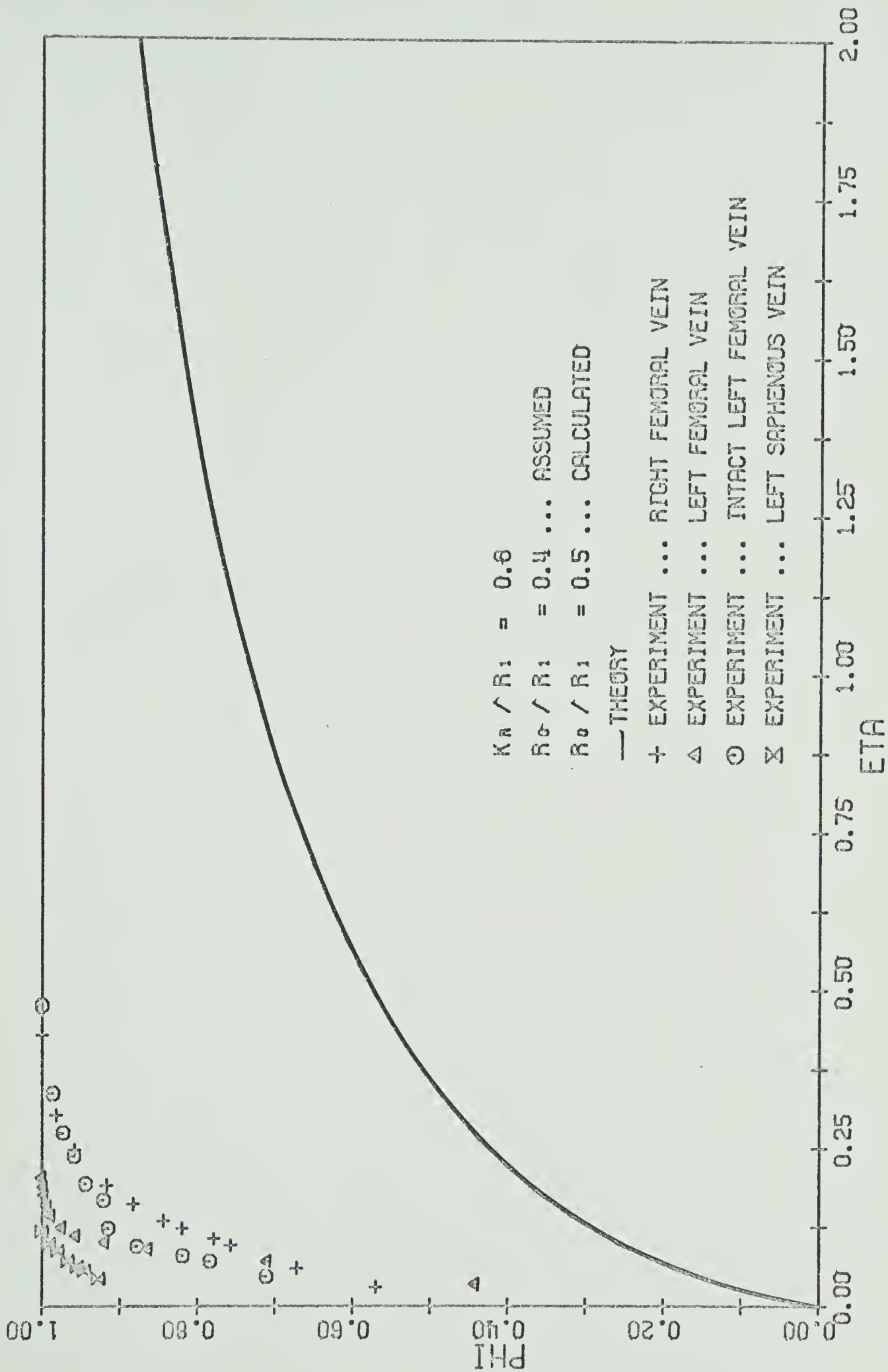


FIG. 1.2 PHYSIOLOGICAL EXPERIMENTS; ASSUMING $R_0/R_1 = 0.4$

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